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Refined mechanical and mathematical model of an elastic half-plane

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Abstract. Loads cause vertical movements of the foundations of all the structures. Their magnitude determines the building safe operation. A closed analytical solution to the problem of the linear elastic theory for the distribution of stresses and strains in a homogeneous isotropic elastic foundation has been presented. The article considers the calculation of the stress-strain state of an elastic half-plane by the method in displacement functions. The theory of calculating an elastic half-plane has been built. New formulas have been are found that determine displacements and stresses at any points of an elastic foundation. An example of calculating an elastic half-plane under the action of a normal and tangential loads has been given.
Keywords: elastic foundation, half-plane, displacements, stresses, deformation, distribution function.

1. Introduction

Under the impact of various loads, all the structures under construction undergo greater or lesser vertical displacements (settlements), as well as horizontal shears that must be taken into account when calculating foundation bases. If the settlement values do not exceed some predetermined value, then it is considered that the long-term safe operation of the structure is ensured. In this regard, the calculation of the foundations of structures by deformations (according to the second group of limit states) is one of the most important problems of soil mechanics.

Numerous experiments have established [1] that deformations of soils under foundations develop mainly in the upper zone of the foundation, therefore, to analyze the stress-strain state of the foundations of structures, it is possible to use the calculation models based on solutions of the elastic theory [2,3].

The method of complex potentials has been used to solve a number of topical problems in the mechanics of a deformable solid body [4–6], as well as mining mechanics and soil mechanics [7,8].

Work provides a solution to the problem of stress distribution in the soil massif with a uniform displacement of the boundary section of the elastic half-plane, which was used to calculate the total settlement of the strip foundation, taking into account additional stresses arising in the soil massif due to the displacement of the loaded boundary section.

However, in practice, uneven movements are often observed that lead, for example, to the occurrence of the structure rolls. In works [9], the problems of the stress-strain state of a soil massif with linear displacement of a section of its boundary that simulates this type of the structure movement, are considered.

Andresen et al. [10] considered a means of determining the total effective bulk stress. Half-planar contacts subjected to a stressed state are considered in the following works [11-13]. Comprehensive studies of the stress-strain state of an isotropic half-plane with cracks are shown in the works [14-17].

In this article, within the framework of the model of a linearly deformable medium, the problem of the stress-strain state of an elastic half-plane is considered. The solution of the problem has been performed by the method in displacement functions.

2. Methods

To obtain the mechanical and mathematical model of an elastic half-plane, we use the method in displacement functions in solving elementary problems of the two-dimensional theory of elasticity. The method for determining the stress-strain state of an elastic half-plane makes it possible to obtain solutions to problems of the plane theory of elasticity not only for stresses, but also for displacements. Differential dependences of the stress and displacement components make it possible to obtain resolving equations for solving a specific problem.

Let's imagine an elastic foundation in the form of a half-plane and let's consider it in the Cartesian coordinate system. The stress-strain state of the elastic foundation will be determined by the calculation method in displacement functions.

To obtain a mathematical model, let's use the basic relations of the plane elastic theory [18].

Stress balance Eq. (1):

$$\frac{\partial \sigma_1}{\partial x_1} + \frac{\partial \tau_{13}}{\partial x_3} = 0,$$

$$\frac{\partial \tau_{13}}{\partial x_1} + \frac{\partial \sigma_3}{\partial x_3} = 0. \tag{1}$$

Strain components (Cauchy) Eq. (2):

$$\varepsilon_1 = \frac{\partial U_1}{\partial x_1}, \quad \varepsilon_3 = \frac{\partial U_3}{\partial x_3}, \quad \gamma_{13} = \frac{\partial U_1}{\partial x_3} + \frac{\partial U_3}{\partial x_1}. \tag{2}$$

Physical relationships (Hooke law) Eq. (3):

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - v\sigma_3), \quad \varepsilon_3 = \frac{1}{E}(\sigma_3 - v\sigma_1), \quad \gamma_{13} = \frac{\tau_{13}}{G}, \tag{3}$$

$$G = \frac{E}{2(1+v)},$$

where: σ_1, σ_3 are the components of normal stresses along the (x_1, x_3) axes; τ_{13} is a tangential stress; U_1, U_3 are the components of displacements in the direction of coordinate axes (x_1, x_3); $\varepsilon_1, \varepsilon_3$ are linear deformations; γ_{13} is the shear deformation; E, G, v are the elasticity modulus, the shear modulus and the Poisson coefficient of the elastic foundation material.

Based on Eq. (2), the components of stresses Eq. (3) will take the form Eq. (4):

$$\sigma_1 = \overline{E}\left(\frac{\partial U_1}{\partial x_1} + v\frac{\partial U_3}{\partial x_3}\right),$$

$$\sigma_3 = \overline{E}\left(\frac{\partial U_3}{\partial x_3} + v\frac{\partial U_1}{\partial x_1}\right), \tag{4}$$

$$\tau_{13} = G\left(\frac{\partial U_1}{\partial x_3} + \frac{\partial U_3}{\partial x_1}\right),$$

where: $\overline{E} = \dfrac{E}{1 - v^2}$ is the generalized elasticity modulus.

Substituting Eq. (4) into the first equation of system Eq. (1), we obtain the balance equation relative to displacements:

$$\left[\frac{\overline{E}}{G}\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}\right](U_1) + \left(1 + v\frac{\overline{E}}{G}\right)\frac{\partial^2}{\partial x_1 \partial x_3}(U_3) = 0.$$

By expressing the components of displacements through the function of displacements F, let's determined the solution as follows Eq. (5):

$$U_3 = \frac{\overline{E}}{G}\frac{\partial^2 F}{\partial x_1^2} + \frac{\partial^2 F}{\partial x_3^2},$$

$$U_1 = -\left(1 + v\frac{\overline{E}}{G}\right)\frac{\partial^2 F}{\partial x_1 \partial x_3},$$

(5)

where: $F(x_1, x_3)$ is the function of displacement.

Substituting stresses Eq. (4) into the second expression of system Eq. (1), we obtain:

$$\left(1 + v\frac{\overline{E}}{G}\right)\frac{\partial^2 U_1}{\partial x_1 \partial x_3} + \left[\frac{\partial^2}{\partial x_1^2} + \frac{\overline{E}}{G}\frac{\partial^2}{\partial x_3^2}\right](U_3) = 0.$$

Substituting Eq. (5) into this equation, let's determine the resolving equation for F:

$$\frac{\partial^4 F}{\partial x_1^4} + \left[-2v + \left(1 - v^2\right)\frac{\overline{E}}{G}\right]\frac{\partial^4 F}{\partial x_1^2 \partial x_3^2} + \frac{\partial^4 F}{\partial x_3^4} = 0.$$

Taking into consideration the $\overline{E}$ and G values, we obtain the equation in the standard form Eq. (6):

$$\nabla^2 \nabla^2 F = \frac{\partial^4 F}{\partial x_1^4} + 2\frac{\partial^4 F}{\partial x_1^2 \partial x_3^2} + \frac{\partial^4 F}{\partial x_3^4} = 0.$$

(6)

Based on Eq. (5), components of stresses Eq. (4) will be as follows Eq. (7):

$$\sigma_1 = -\overline{E}\frac{\partial}{\partial x_3}\left(\frac{\partial^2 F}{\partial x_1^2} - v\frac{\partial^2 F}{\partial x_3^2}\right),$$

$$\sigma_3 = \overline{E}\frac{\partial}{\partial x_3}\left(\frac{\partial^2 F}{\partial x_3^2} + (2 + v)\frac{\partial^2 F}{\partial x_1^2}\right),$$

(7)

$$\tau_{13} = \overline{E}\frac{\partial}{\partial x_1}\left(\frac{\partial^2 F}{\partial x_1^2} - v\frac{\partial^2 F}{\partial x_3^2}\right).$$

In order to solve biharmonic Eq. (6), let's represent the function of displacement in the following form [19] Eq. (8):

$$F(x_1, x_3) = \delta(x_3) \cdot W(x_1),$$

(8)

where: $\delta(x_3)$ is the distribution function; $W(x_1)$ is the flexure function.

Taking into consideration function of displacement Eq. (8) and the equation of transitional processes $\dfrac{d^2W(x_1)}{dx_1^2} = -\bar{k}^2 W(x_1); \quad \dfrac{d^4W(x_1)}{dx_1^4} = \bar{k}^4 W(x_1)$, let's find components of displacements Eq. (5) and components of stresses Eq. (7):

$$U_1 = -\left(1 + \nu \frac{E}{1-\nu^2}\frac{2(1+\nu)}{E}\right)\cdot \delta'(x_3)\frac{dW(x_1)}{dx_1} = -h\left(\frac{1+\nu}{1-\nu}\right)\cdot \delta'(z_0)\frac{dW(x_1)}{dx_1},$$

$$U_3 = \frac{E}{1-\nu^2}\frac{2(1+\nu)}{E}\cdot \frac{d^2W(x_1)}{dx_1^2}\cdot \delta(x_3) + W(x_1)\cdot \delta''(x_3) = \left[-\frac{2\bar{k}^2 h^2}{1-\nu}\cdot \delta(z_0) + \delta''(z_0)\right]\cdot W(x_1),$$

$$\sigma_1 = -\bar{E}\frac{\partial}{\partial x_3}\left(\delta(x_3)\frac{d^2W(x_1)}{dx_1^2} - \nu\cdot W(x_1)\delta''(x_3)\right) = -\frac{12\bar{E}h}{12}\left[\delta'(z_0) + \frac{\nu}{\bar{k}^2 h^2}\delta'''(z_0)\right]\frac{d^2W(x_1)}{dx_1^2},$$

$$\sigma_3 = \bar{E}\frac{\partial}{\partial x_3}\left[\delta''(x_3)\cdot W(x_1) + (2+\nu)\cdot \delta(x_3)\frac{d^2W(x_1)}{dx_1^2}\right] =$$

$$= \frac{12\bar{E}h^3}{12\bar{k}^4 h^4}\left[\delta'''(z_0) - (2+\nu)\cdot \bar{k}^2 h^2 \delta'(z_0)\right]\frac{d^4W(x_1)}{dx_1^4},$$

$$\tau_{13} = \bar{E}\frac{\partial}{\partial x_1}\left(\delta(x_3)\frac{d^2W(x_1)}{dx_1^2} - \nu\cdot W(x_1)\delta''(x_3)\right) = \frac{12\bar{E}h^2}{12}\left[\delta(z_0) + \frac{\nu}{\bar{k}^2 h^2}\delta''(z_0)\right]\frac{d^3W(x_1)}{dx_1^3}.$$

Based on these equations, we obtain Eq. (9) and Eq. (10):

$$U_1(x_1, x_3) = -h\cdot \varphi(z_0)\frac{dW(x_1)}{dx_1},$$

$$U_3(x_1, x_3) = f(z_0)\cdot W(x_1),$$

$$\varphi(z_0) = \frac{(1+\nu)}{(1-\nu)}\delta'(z_0), \quad f(z_0) = \delta''(z_0) - \frac{2}{(1-\nu)}k^2\delta(z_0), \tag{9}$$

$$z_0 = \frac{x_3}{h}; \quad k^2 = \bar{k}^2\cdot h^2,$$

where: h is the elastic foundation thickness; z_0 is a dimensionless transversal coordinate; $\varphi(z_0), f(z_0)$ is the function of displacement distribution (U_1, U_3); $\bar{k}$ is the strained foundation parameter.

$$\sigma_1 = -\frac{\bar{E}h}{12}\cdot \psi'(z_0)\frac{d^2W(x_1)}{dx_1^2},$$

$$\tau_{13} = \frac{\bar{E}h^2}{12}\cdot \psi(z_0)\frac{d^3W(x_1)}{dx_1^3},$$

$$\sigma_3 = \frac{\bar{E}h^3}{12}\cdot \alpha(z_0)\frac{d^4W(x_1)}{dx_1^4}, \tag{10}$$

$$\psi(z_0) = 12\left[\delta(z_0) + \frac{\nu}{k^2}\delta''(z_0)\right], \quad \alpha(z_0) = \frac{12}{k^4}\left[\delta'''(z_0) - (2+\nu)k^2\delta'(z_0)\right],$$

$$z_0 = \frac{x_3}{h}; \quad k^4 = \bar{k}^4 \cdot h^4,$$

where: $\psi'(z_0), \psi(z_0), \alpha(z_0)$ is the function of stress distribution $\sigma_1, \tau_{13}, \sigma_3$.

Substituting into resolving Eq. (6) function of displacement Eq. (8) and taking into account the equation of transitional processes, we obtain:

$$\delta(x_3) \cdot \frac{d^4 W(x_1)}{dx_1^4} + 2\delta''(x_3) \cdot \frac{d^2 W(x_1)}{dx_1^2} + \delta^{IV}(x_3) \cdot W(x_1) = 0,$$

$$\left[\delta^{IV}(z_0) - 2 \cdot \bar{k}^2 h^2 \delta''(z_0) + \bar{k}^4 h^4 \delta(z_0) \right] \cdot W(x_1) = 0,$$

$$\delta^{IV}(z_0) - 2 \cdot k^2 \delta''(z_0) + k^4 \delta(z_0) = 0.$$

Its general solution is written down in the following form:

$$\delta(z_0) = (C_1 + C_2 z_0) e^{-kz_0} + (C_3 + C_4 z_0) e^{kz_0} = (A_1 + A_2 z_0) chkz_0 + (A_3 + A_4 z_0) shkz_0.$$

If to use the general solution in the elastic half-plane $(z_0 \to \infty, \ \delta(z_0) = 0)$, then this solution will take the form Eq. (11):

$$\delta(z_0) = (C_1 + C_2 z_0) e^{-kz_0}. \tag{11}$$

Thus, if the functions $W(x_1)$ and $\delta(z_0)$, are known, then the proposed method of calculation makes it possible to obtain the stress-strain state for the elastic half-plane.

3. Results

Similar problems (rigid punch, semi-infinite plane) were considered by Sadovsky for a punch and by Flaman (action of a concentrated force). These problems have the following features: it is impossible to determine the values of the reactive pressure at the corner points of the stamp; it is impossible to determine the vertical displacement under the force. These shortcomings in the considered problems are easily eliminated by applying the method in displacement functions.

Example. Let a normal $\sigma(x_1)$ and tangential $\tau(x_1)$ distributed loads act on the elastic half-plane $(z_0 = 0)$. Based on Eq. (10), we write the boundary conditions in the form Eq. (12):

$$\tau_{13} = \tau: \ \beta_0 \frac{\bar{E}h^2}{12} \frac{d^3 W(x_1)}{dx_1^3} = \tau(x_1); \quad \psi(0) = \beta_0,$$

$$\sigma_3 = \sigma: \ \alpha_0 \frac{\bar{E}h^3}{12} \frac{d^4 W(x_1)}{dx_1^4} = \sigma(x_1); \quad \alpha(0) = \alpha_0, \tag{12}$$

where: α_0, β_0 are the parameters of the normal and tangential loads.

Let's consider some private cases.

1) If the normal load $\sigma(x_1) = \sigma_0 \sin \frac{\pi x_1}{L}$ act on an elastic half-plane, then $\beta_0 = 0, \ \alpha_0 \neq 0$, based on Eq. (12):

$$\alpha_0 \frac{\bar{E}h^3}{12} \frac{d^4 W(x_1)}{dx_1^4} = \sigma_0 \sin \frac{\pi x_1}{L}.$$

By integrating the equation, we obtain the flexure function Eq. (13):

$$W(x_1) = W_0 \sin \frac{\pi x_1}{L}; \quad W_0 = \frac{\sigma_0 L^4}{\alpha_0 \pi^4 E J_0}, \quad J_0 = \frac{h^3}{12}. \tag{13}$$

2) If only a tangential load $\tau(x_1) = \tau_0 \cos \frac{\pi x_1}{L}$ acts on an elastic half-plane, then $\beta_0 \neq 0$, $\alpha_0 = 0$, based on Eq. (12):

$$\beta_0 \frac{\overline{E} h^2}{12} \frac{d^3 W(x_1)}{dx_1^3} = \tau_0 \cos \frac{\pi x_1}{L}.$$

By integrating we find the flexure function Eq. (14):

$$W(x_1) = -W_0 \sin \frac{\pi x_1}{L}, \quad W_0 = \frac{\tau_0}{\beta_0 \pi^3} \cdot \frac{L^3}{E J_1}, \quad J_1 = \frac{h^2}{12}. \tag{14}$$

In the general case $\beta_0 \neq 0$, $\alpha_0 \neq 0$, the, taking into consideration Eq. (10), boundary conditions Eq. (12) will be represented in the following form Eq. (15):

$$\psi(0) = \beta_0 : \quad \frac{\beta_0}{12} = \delta(0) + \frac{v}{k^2} \delta''(0),$$

$$\alpha(0) = \alpha_0 : \quad \frac{k^4}{12} \alpha_0 = \delta'''(0) - (2+v) k^2 \delta'(0). \tag{15}$$

Let's determine function Eq. (11) and the derivatives at the $(z_0 = 0)$ point Eq. (16):

$$\delta'(z_0) = (-k C_1 + C_2 - k z_0 C_2) \cdot e^{-k z_0},$$
$$\delta''(z_0) = (-2k C_2 + k^2 C_1 + k^2 z_0 C_2) \cdot e^{-k z_0},$$
$$\delta'''(z_0) = (-k^3 C_1 + 3k^2 C_2 - k^3 z_0 C_2) \cdot e^{-k z_0}, \tag{16}$$
$$\delta(0) = C_1, \quad \delta'(0) = -k C_1 + C_2,$$
$$\delta''(0) = k^2 C_1 - 2k C_2, \quad \delta'''(0) = 3k^2 C_2 - k^3 C_1.$$

Substituting these values into Eq. (15), we obtain the following system of equations:

$$\frac{\beta_0}{12} = C_1(1+v) - \frac{2v}{k} C_2,$$

$$\frac{k}{12} \alpha_0 = C_1(1+v) + \frac{(1-v)}{k} C_2.$$

From these equations there are determined the constants Eq. (17):

$$C_1 = \frac{1}{12(1+v)^2} [(1-v)\beta_0 + 2kv\alpha_0],$$

$$C_2 = \frac{k}{12(1+v)} [k \cdot \alpha_0 - \beta_0]. \tag{17}$$

Substituting the C_1, C_2 values into Eq. (11), we determine the solution of the $\delta(z_0)$ function Eq. (18):

$$\delta(z_0) = \left\{ \frac{1}{12(1+v)^2}\left[(1-v)\beta_0 + 2kv\alpha_0\right] + \frac{k}{12(1+v)}\left[k\alpha_0 - \beta_0\right]z_0 \right\} e^{-kz_0} . \tag{18}$$

To determine the components of displacements and stresses, we set the numerical data of the parameters. The parameters of the normal and tangential loads are $\beta_0 = 1$, $\alpha_0 = 1$, the length of the elastic half-plane is $L = 10$ m, the thickness of the elastic half-plane is $h = 3$ m, the elasticity modulus of the elastic half-plane is $\overline{E} = 30$ Pa.

The solutions of the problem have been obtained using the computer program MathCad. Figure 1 shows the flexure function of the elastic half-plane at $\overline{E} = 10, 20, 30$ Pa. The results of displacements, normal and shear stresses of the elastic half-plane are shown in Figures 2-4.
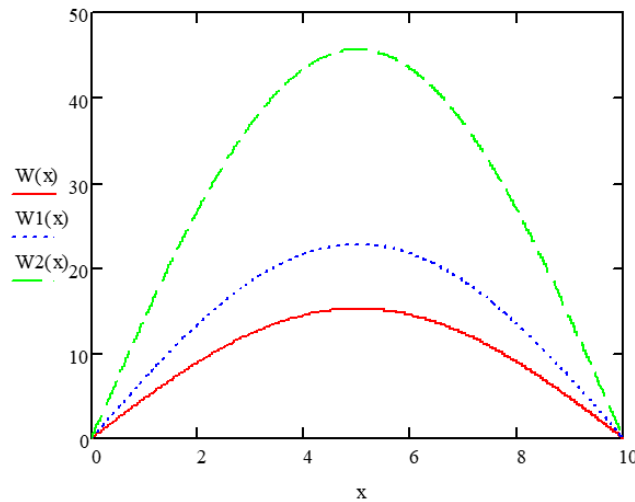
Figure 1 – The flexure function of the elastic half-plane
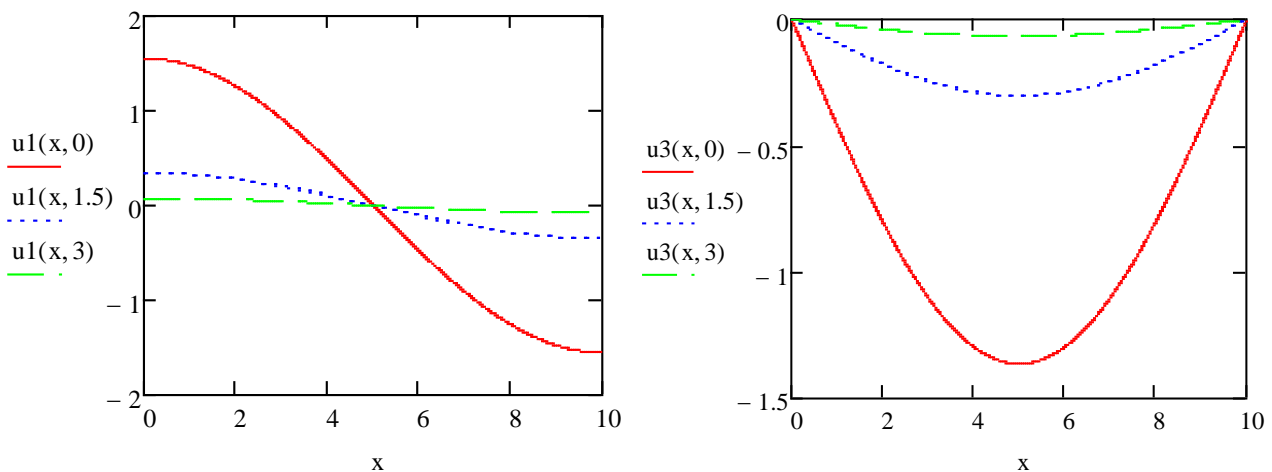
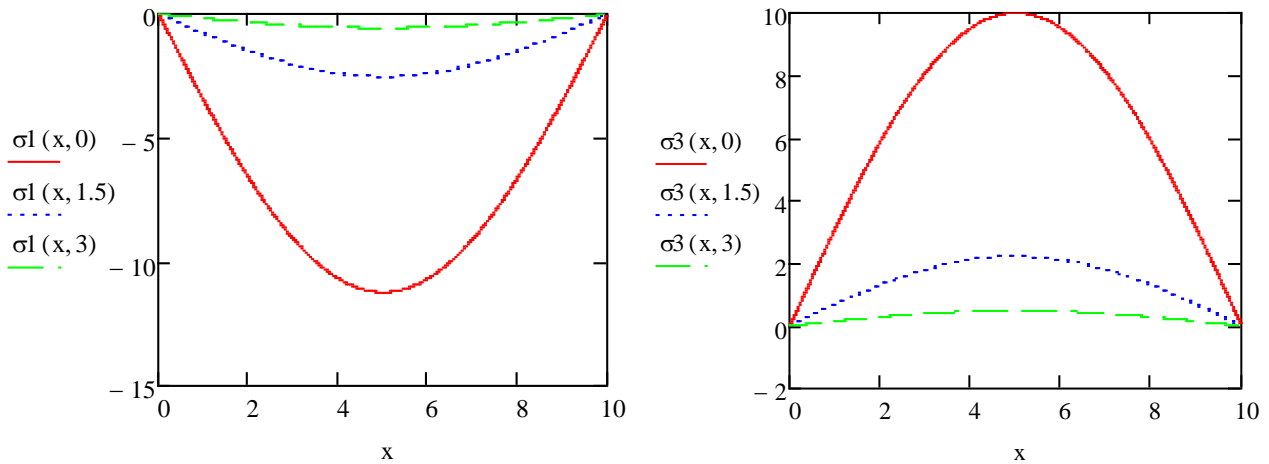Figure 2 – The components of displacements

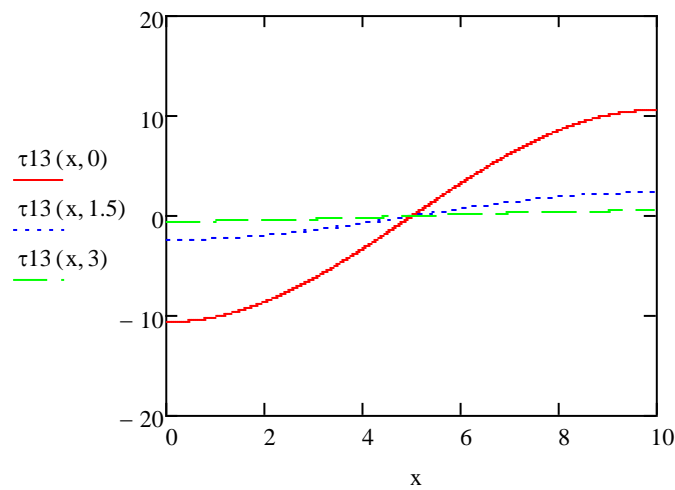Figure 3 – The components of normal stresses

Figure 4 – A tangential stress

As can be seen from the Figures 1-4, with an increase in the thickness of the elastic foundation, the values of the components of displacements and stresses decrease. It is known that with an increase in the thickness of the elastic base, the values of the components of displacements and stresses are not taken into account.

Thus, the results obtained allow determining displacements Eq. (9) and stresses Eq. (10) of the elastic half-plane in an analytical form.

4. Conclusions

Summarizing the obtained results, we can draw the following conclusions:
1. The theory for calculating the elastic half-plane has been developed.
2. The stress and displacement components of the elastic half-plane have been obtained in a closed form.
3. The distribution functions of displacements and stresses have been found in analytical form.
4. The flexure function of the elastic half-plane has been obtained.

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