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Article

Calculation and numerical modeling of the effect of heat and mass transfer on the properties of pile foundations in seasonally freezing soils

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Abstract. The phenomenon of heat and mass transfer in seasonally freezing soils has a significant impact on the condition of pile foundations. Building structures constructed in such soils experience extremely negative consequences: frost heave, vibration dynamics, moisture mass transfer and soil weakening during the thawing. Seasonally freezing soils occupy most of the territory of Kazakhstan, and in most regions the depth of soil freezing exceeds 1.5 m, there are settlements where the index reaches 1.97 m (Semiyarka, East Kazakhstan region) and in the north of the country - 2.74 m (Arshaly, Akmola region). Arrangement of foundation bases below the frost depth leads to increased construction costs, requires additional costs for thermal insulation, ventilation, other materials and structures, and, in addition, does not always lead to a full levelling of the negative impact of heat and mass transfer on the term and conditions of operation of buildings, structures, railways and roads. Therefore, the efforts of engineers and specialists in the field of thermal physics are aimed at finding effective ways to solve the problems of deformation and destruction of foundations in seasonally freezing soils. In the article an attempt is made to reveal the character of heat and mass transfer influence on pile foundations in seasonally freezing soils on the basis of a thermomechanical model.
Key words: heat and mass transfer, pile foundations, seasonally freezing soils, frost heave deformations, thawing, freezing of soils.

1. Introduction

The relevance of the chosen topic of the study is due to the fact that seasonal freezing of soils is observed in the territory occupying more than half of the total area of Kazakhstan. Foundations in such soils are exposed to the influence of processes: freezing, frost heaving, thawing and vibrodynamics. These processes are especially intensive during seasonal thawing and freezing. The large extent of the territory of Kazakhstan from west to east and from north to south, the complexity of geographical, geological, tectonic, hydrogeological structure, landscape and climatic conditions have largely predetermined the peculiarity of seasonal freezing of soils in different regions of our country. Therefore, the main regularities of seasonal freezing at a particular site are determined by the conditions of heat exchange on the soil surface, its composition and condition. Figure 1 shows a schematic map of the maximum depth of penetration of the zero isotherm into the ground for Kazakhstan.

Pile foundations on seasonally freezing soils require a numerical study of the temperature regime, taking into account the processes of heat and mass transfer with phase transitions in order to determine their bearing capacity and stability to avoid further problems with the operation of buildings and structures [1].

The purpose of the article is to study the effect and patterns of heat and mass transfer on pile foundations in seasonally freezing soils.

The phenomenon of heat and mass transfer is actively investigated in thermophysics, poromechanics and thermohydrodynamics. Migration processes occurring in thawing and freezing soils have been studied by: E.D. Ershov, A.I. Korotkiy, S. Krauch, O. Coussy, S.A. Kudryavtsev.

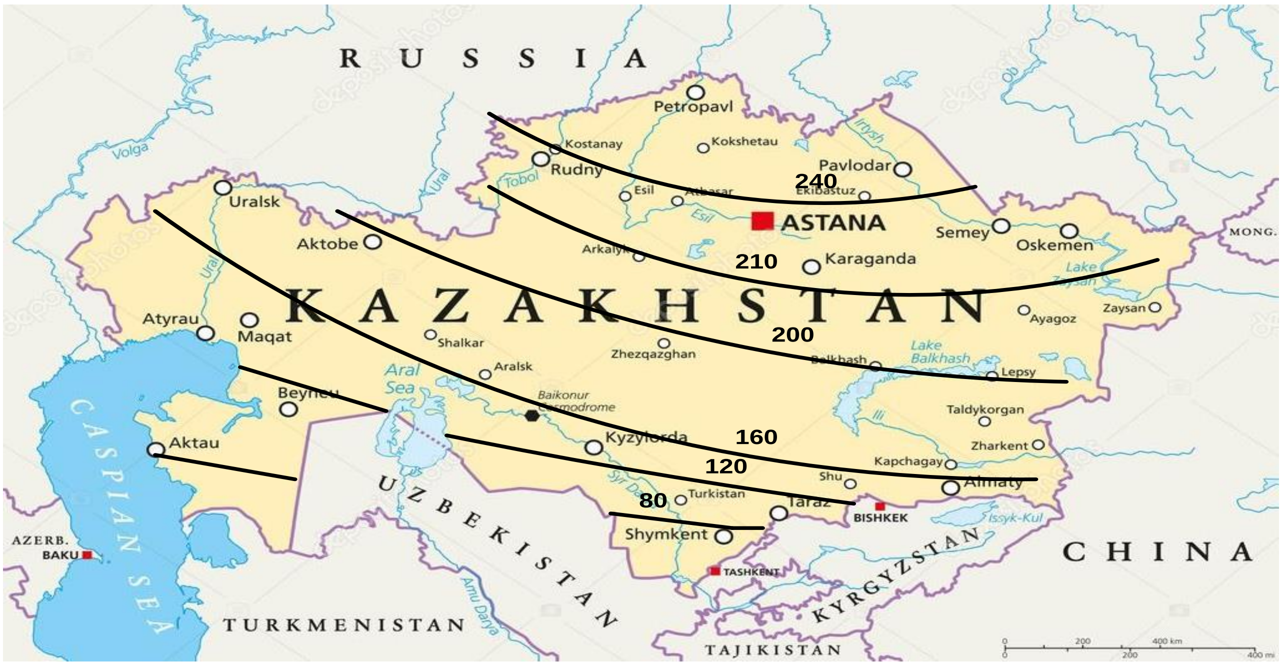

Figure 1 – Schematic map of the maximum depth of penetration of the zero isotherm into the ground [2]

Numerical and analytical methods for solving problems of heat and mass transfer in seasonally freezing soils are presented in the works of V.I. Popov, D.S. Skvortsov, E.M. Kartashov, A.Yu. Krainov, etc.

It should be noted that mathematical modelling of only the heat conduction process with phase transformation requires rather time-consuming iterative procedures. In the case of joint heat and mass transfer, the situation is aggravated and more than 150-300 iterations are required for joint solution of the mass and heat transfer equations in the known, at each time step, range of their variation. The reason for this is the incomplete use of those possibilities, which are determined by the phase equilibrium equation, linking the state parameters (temperature, moisture content, rock porosity, salt concentration in brine) satisfying the transport equations [3].

There are successfully tested numerical methods for approximate calculations of heat and mass transfer in soils (V.I. Popov, O.V. Tretyakova, etc.)[19]. In addition, today the solution of the heat and mass transfer problem by numerical and analytical methods is significantly facilitated by the use of special computer programmes, among which we can mention Frost Termo, Borey 3D, FrozenWall, Termoground. These programs allow to perform thermal and hydrodynamic calculations and provide opportunities for visual modelling of soil temperature fields and construction of necessary graphs and diagrams.

2. Methods

Laboratory methods of research were carried out, as a result of which we determined the soil filtration coefficient for Quaternary loams at the level of 0.01-0.13 m/day. Groundwater at this road section is characterised as sodium chloride, very hard, slightly alkaline, brackish. In relation to concrete of W4 grade on Portland cement, ground waters are non-aggressive and slightly aggressive, in relation to reinforced concrete structures they are moderately aggressive. Corrosive aggressiveness of ground water towards lead and aluminium cable sheathing is high. In relation to steel structures (according to Stabler) ground waters are corrosive. The main physical characteristics of the foundation soils are given in Table 1.

Table 1 – Results of laboratory analyses of foundation soils

Name of indicators	Unit	Value
Natural humidity	%	20.2
Liquid limit	%	26.5
Humidity at rolling limit	%	15.1
Plastisity index	%	11.6
Liquidity index		0.5
Soil density	g/cm³	2.00
Particle density of soil	g/cm³	2.70
Porosity coefficient	доли ед.	0.700
Moisture content	доли ед.	0.800
Deformation modulus	МПа	6.50
Specific cohesion	кПа	23.5
Angle of internal friction	degree	22

Partial values of strength and deformation properties of quaternary loams were subjected to static processing in accordance with the requirements of GOST 20522-2012 and as a result normative and calculated values of characteristics given in the table were obtained. The deformation modulus is 6.50 MPa. It is recommended to take the value equal to 6.0 MPa as the design value of the modulus of deformation.

The analysis of mechanical properties of foundation soils is given in Table 2.

Table 2 – Mechanical and deformation characteristics of foundation soils

Characteristics name	Unit	Values	on deformations α=0,85	for load capacity α=0,95
Specific cohesion	kPa	23.5	13.0	6.5
Angle of internal friction	degree	22	21	20
Deformation modulus	MPa	6.50	6.00	-
Soil density	g/cm³	2.00	1.98	1.97

To study the effect of heat and mass transfer on pile foundations in seasonally freezing soils, a thermohydromechanical model of soil subjected to freezing and thawing processes was used.

This model assumes that the soil is a three-phase porous medium consisting of soil particles, liquid water and ice.

The thermohydromechanical model includes a system of nonlinear equations consisting of mass transfer, heat transfer and equilibrium equations expressing such fundamental laws of continuum mechanics as the laws of conservation of mass, energy and momentum.

Important relations in the model are Bishop's formula for calculating the pore pressure in the freezing ground zone and the Clausius-Claiperon equation, which describes the intensity of cryogenic suction as a function of temperature [4].

The application of numerical modelling methods for the behaviour of pile foundations makes it possible to control by stages the processes of changes in temperature and moisture fields and associated deformations, and, consequently, to predict the efficiency of using various materials and measures to reduce or eliminate the negative phenomena acting on foundations and subgrade soils in conditions of their seasonal freezing-thawing [5].

We proceed from the assumption that seasonally freezing ground is a three-phase porous medium consisting of solid particles (index s), liquid water (index l) and ice crystals (index i).

In accordance with the existing ideas about the process of ground freezing, the following hypotheses are accepted for the construction of the thermohydromechanical model:

- At the initial moment of time the porous medium is completely saturated with water;
- The rheological properties of porous medium under long-term loading are described by viscoelastic deformation [6].

2.1 Mass transfer equation

In accordance with the law of conservation of water and ice mass, the equation of mass transfer taking into account the phase transition of pore water into ice can be written as Eq. (1):

$$\frac{\partial(\rho_j S_j n)}{\partial t}+\frac{\partial(\rho_i S_i n)}{\partial t}+div(\rho_l v_l)=0 \tag{1}$$

Where: ρSn – mass water content ($j=l$) and ice ($j=i$) at a point in time t; ρ – phase density; S– phase saturation; n– porosity; v_l– water velocity relative to the solid skeleton.

Ice saturation $S_i$is given by a step function of temperature in Eq. (2):

$$S_i=\begin{cases}1-\left[1-\left(T-T_{ph}\right)\right]^{\alpha}, & T\leq T_{ph}\\ 0, & T>T_{ph},\end{cases} \tag{2}$$

Where: T_{ph} – the freezing point of water; α – experimental parameter.

Moisture saturation $S_l$is determined from the condition of complete saturation of the porous medium by the Eq. (3):

$$S_l=1-S_i. \tag{3}$$

The velocity of moisture movement v_l relative to soil particles is determined by Darcy's law by Eq. (4):

$$v_l=-k\cdot grad\psi, \tag{4}$$

Where: k– moisture conductivity coefficient; ψ – groundwater potential.

The moisture conductivity coefficient is calculated as a function of temperature in Eq. (5):

$$k=\begin{cases}k_0\left[1-\left(T-T_{ph}\right)^{\beta}\right], & T\leq T_{ph}\\ k_0, & T>T_{ph}\end{cases} \tag{5}$$

Where: k_0 – moisture conductivity coefficient of unfrozen soil; β – experimental parameter [7].

Soil moisture potential is determined by the Eq. (6):

$$\psi=\frac{p_l}{\rho_l g}+z \tag{6}$$

Where: p_l – pore moisture pressure; g – free-fall acceleration; z– vertical coordinate.

Bishop's relationship is used to determine pore pressure by Eq. (7):

$$p=\chi p_l+(1-\chi)p_i, \tag{7}$$

where p_i – ice pore pressure; χ – coefficient depending on ice saturation.

The coefficient χ is determined by the Eq. (8):

$$\chi=(1-S_i)^{1.5}. \tag{8}$$

The ice pressure p_i is determined from the Clausius-Claiperon equation according to the expression in Eq. (9):

$$p_i=\frac{(\rho_l-\rho_i)p_0+\rho_i\rho_l L\ln\left(\frac{T}{T_{ph}}\right)-\rho_i p_l}{\rho_l} \tag{9}$$

where L– specific heat of crystallisation of water; p_0 – initial pore pressure.

From Eqs. (7) and (9) it follows that water pressure $p_l$can be calculated as a function of pore pressure $p$and temperature T by Eq. (10).

It follows from relations in Eqs. (7) and (9) that water pressure p_l can be calculated as a function of pore pressure and temperature using Eq. (10):

$$p_l=\frac{(1-\chi)(\rho_l-\rho_i)p_0+(1-\chi)\rho_i\rho_l L\ln\left(\frac{T}{T_{ph}}\right)+\rho_l p}{\chi\rho_l+(1-\chi)\rho_l} \tag{10}$$

2.2 Heat transfer equation

The heat transfer equation is derived from the law of conservation of energy, taking into account heat transfer due to the mechanisms of heat conduction and convection, as well as heat release during water crystallisation [8]. Then the heat transfer equation is written in the Eq. (11):

$$C\frac{\partial T}{\partial t}-div(\lambda gradT)+C_l v_l\cdot gradT=Q_{ph} \tag{11}$$

Where: C – volumetric heat capacity; λ – heat transfer coefficient of three-phase porous medium; C_l – volumetric heat capacity of water; Q_{ph} – heat source associated with the latent heat of the phase transition of water to ice.

The heat source Q_{ph} is determined by the Eq. (12):

$$Q_{ph} = L\rho_i \frac{\partial(nS_i)}{\partial t} \tag{12}$$

The volumetric heat capacity is calculated on the basis of the mixture rule according to the Eq. (13):

$$C = (1-n)\rho_s c_s + n(S_l\rho_l c_l + S_i\rho_i c_i) \tag{13}$$

Where: c_j – is the specific heat capacity of the phase at constant pressure $j(j = s, l, i)$.

The heat transfer coefficient is calculated on the basis of the ratio for the stepped average as in Eq. (14):

$$\lambda = \lambda_s^{1-n}\lambda_i^{nS_i}\lambda_l^{nS_l} \tag{14}$$

Where: λ_j – phase heat transfer coefficient $j(j = s, l, i)$.

2.3 Equilibrium equation

It follows from the laws of poromechanics and momentum conservation that the equilibrium equation for a saturated porous medium can be written with respect to the total stress tensor σ in the Eq. (15)

$$div\sigma + \gamma = 0 \tag{15}$$

Where: γ – specific gravity of porous medium.

The specific gravity of the porous medium is determined by the Eq. (16):

$$\gamma = [(1-n)\rho_s + n(S_l\rho_l + S_i\rho_ш)]g \tag{16}$$

Where: ρ_s– soil particle density.

In turn, the total stress tensor can be through the effective stress σ tensor of the solid skeleton $\sigma^{'}$and the pore pressure p by the Eq. (17)

$$\sigma = \sigma^{'} - bpI \tag{17}$$

Where: I – unit tensor; b – effective Bio coefficient.

The effective stress tensor $\sigma^{'}$ is determined on the basis of Hooke's law for linear-elastic isotropic material from the relation in Eq. (18):

$$\sigma^{'} = \left(K - \frac{2}{3}G\right)\varepsilon_{vol}^{el}I + 2G\varepsilon^{el} \tag{18}$$

Where: K – effective bulk modulus; G – effective shear modulus; ε^{el} – elastic strain tensor; ε_{vol}^{el} – volume elastic deformation value.

According to the principle of additive decomposition of the total strain tensor ε, the elastic strain tensor $\varepsilon^{el}$is defined by the Eq. (19):

$$\varepsilon^{el} = \varepsilon - \varepsilon^{th} - \varepsilon^{in} \tag{19}$$

Where: ε^{th} – temperature deformation tensor.

The total deformation tensor ε is defined from the geometrical relation for small deformations in the Eq. (20):

$$\varepsilon = \frac{1}{2}(grad(u) + grad(u)^T) \tag{20}$$

Where: u – displacement vector.

The temperature deformation tensor is defined by the Eq. (21):

$$\varepsilon^{th} = \alpha_T(T - T_0)I \tag{21}$$

Where: α_T – thermal expansion coefficient; T_0 – initial temperature of unfrozen ground.

The calculation of effective pore pressure is based on the equation of state by O. Coussy [9] as shown in Eq. (22):

$$p = N\left(n - n_0 - b\varepsilon_{vol}^{el} + 3\alpha_T(b - n_0)(T - T_0)\right) \tag{22}$$

Where: n_0 – initial soil porosity; N – effective tangent modulus Bio.

The effective mechanical parameters are determined by the Eq. (23):

$$X = S_i X_{fr} + S_l X_{un} \tag{23}$$

Where: X – effective parameter value; X_{fr} и X_{un} – parameter values in frozen and unfrozen soil zones, respectively.

Keeping the divergent form of the equations, we obtain their Eq. (24):

$$\begin{cases} \frac{\partial(c\rho T)}{\partial t} = -\frac{\partial}{\partial x} J_Q + L_\rho I_F \\ \frac{\partial w}{\partial t} = -\frac{\partial}{\partial x} J_W - I_F \\ \frac{\partial(wC)}{\partial t} = -\frac{\partial}{\partial x} J_C - k_{зах} C I_F - I_A \end{cases} \tag{24}$$

Where: I_A – ion adsorption index on the soil pore surface; $I_F = -\partial m/\partial t$ – Moisture runoff due to water-ice phase transformations; k_{cap} – salt (ion) capture coefficient.

Flows are defined in accordance with the Eq. (25-27):

$$J_W = -K\frac{\partial w}{\partial x} + K\delta_{CW}\frac{\partial C}{\partial x} - K\delta_{TW}\frac{\partial T}{\partial x} J_Q + V_f = J_W^m + V_f \tag{25}$$

$$J_C = -WD_C\frac{\partial C}{\partial x} + CJ_W = J_C^m + CV_f \tag{26}$$

$$J_Q = -\lambda\frac{\partial T}{\partial x} + c_W \rho_W T J_W = J_W^m + c_W \rho_W T V_f \tag{27}$$

Where: λ, K, D_C, c, ρ – heat conductivity, moisture diffusion, salt diffusion, heat capacity and density coefficients, respectively; V_j– moisture filtration rate in porous material; δ_{TW}, δ_{CW}– coefficients accounting for cross-effects [10].

The solution of the initial heat and mass transfer problem can be represented as a sequential solution of subproblems as follows:

- diffusion - system of Eq. (28):

$$\begin{cases} \frac{\partial(c\rho T)}{\partial t} = -\frac{\partial}{\partial x} J_Q^m \\ \frac{\partial w}{\partial t} = -\frac{\partial}{\partial x} J_W^m \\ \frac{\partial(wC)}{\partial t} = -\frac{\partial}{\partial x} J_C^m \end{cases} \tag{28}$$

– of the phase transition is the system of Eq. (29):

$$\begin{cases} \frac{\partial(c\rho T)}{\partial t} = L_\rho I_F \\ \frac{\partial w}{\partial t} = -I_F \\ \frac{\partial(wC)}{\partial t} = -k_{cap} C I_F \end{cases} \tag{29}$$

Solutions of the mass balance equations from Eq. (29) at each time step of Eq. (30):

$$w_2 = w_1 + \Delta m, \quad C_2 = C_1 + \left(\frac{w_1}{w_2}\right)^{1-k_{cap}} \tag{30}$$

The further course of action is shown in Figure 2.

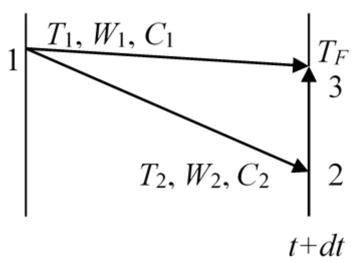

Figure 2 – Diagram of calculation of split heat and mass transfer processes

In Figure 2, transition (1-2), T1 → T2 is determined by the solution of system (28), transition (2-3), T2 → TF is determined by the solution of system (29) [10].

3. Results and Discussion

On the example described in V.I. Vasiliev's thesis "Mathematical modelling of temperature regime of foundation soils in conditions of perennially frozen rocks thermo-hydro-dynamic model" the calculation of heat and mass transfer (Figure 3) in the system "pile - seasonally freezing ground" for the city of Astana was performed [17]:

- Soil parameters: for thawing zone ρ_j = 2,722·106, λ_j = 1,94; for phase transition ρ_L = 125,532·106, δ = 1.0; for freezing zone ρ_i = 2,052·106, λ_j = 2,14 [12];
- The geometrical area contains 20 piles (Figure 3); the pile dimensions are taken: r = 0.2 m, length 14 m;
- Parameters were used as design parameters of the pile system: υT = 1.0, $\rho_p c_\text{p}$ = 1.763·106, R = 0.05;
- Numerical modelling was carried out for the area with dimensions L_X = 15 m, L_y = 54 m, L_Z = 50 m;
- Numerical modelling was carried out for a region of size =15 m, = 54 m, = 50 м;
- The Dirichlet boundary condition with a constant temperature of 20˚C is given;
- Calculations were carried out for t_{max} = 2 years, with step τ = 1 day;
- Initial ground temperature is assumed to be -4˚C;
- Temperature at the upper boundary of the study area is set taking into account the amplitude of air fluctuation in Astana, which, according to SP RK 2.04-01-2017 varies from -51.6 ˚C (absolute minimum) to +41,6 ˚C (absolute maximum) [1].

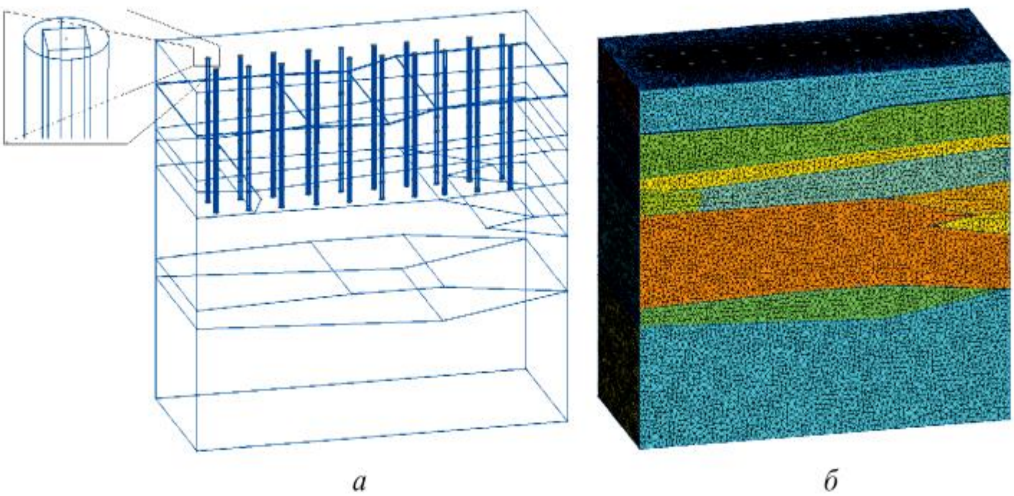

Figure 3 – Computational geometry (a) and tetrahedral mesh (b) for the heat transfer problem in soils taking into account the installation of 20 piles

It should be noted that in applied modeling, even when using significantly non-uniform computational meshes, the dimensionality of the heat transfer problem is quite large. For example, for our heat and mass transfer problem, the computational grid contains 260 thousand nodes, about 1.5 million tetrahedral cells. Numerical solution of such problems is impossible without the use of parallel architecture computing systems [11].

However, the algorithm we use is not associated with source averaging in spatial grid cells and can be generalised to multizone and multidimensional problems. The deviation of the calculation results from the exact solution of the test problem does not exceed 2-2.5%.

Figure 3 shows the results of calculations at a variable temperature of the medium. Inhomogeneous ice accumulation can be explained in different ways. V.I. Popov believes that "... periods of oscillation of the freezing front can be interpreted as its stopping near some average value" [12]. However, it follows from the works of E.D. Ershov[13], S. Crouch [14] and others that the stopping of the front leads to moisture accumulation, as obtained in the present study.

Figure 4 shows the results of exposure of the ground to alternating temperatures with an initial temperature of -4 ºC. A characteristic feature of the result presented in Figure 4 is the slight spatial heterogeneity of ice accumulation. This is determined by the limited feeding area (melting zone) and low diffusion coefficient of moisture in the soil freezing zone.

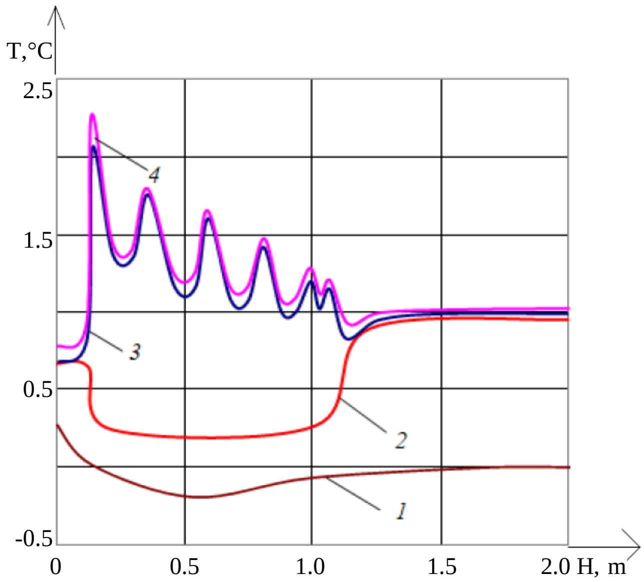

Figure 4 – Simulation results of temperature (1) - (T-273)/ T_n, liquid phase (2) – w/w_0; total moisture content (3) – $(Lod+w)/w_0$and concentration (4) – C/C_0fields

Figure 5 shows the distributions of temperature, moisture content and concentration fields during ground freezing under the influence of alternating temperature of the heat transfer medium for the case k_{cap}= 0.1.

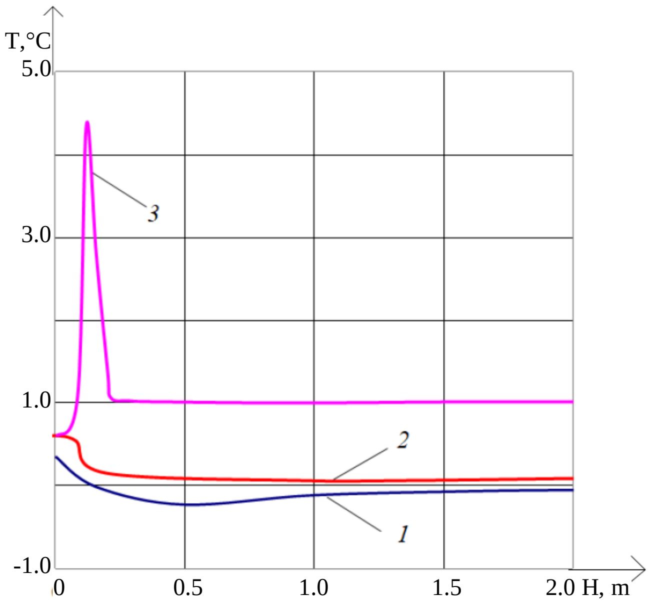

Figure 5 – Distribution of temperature (1), humidity (liquid phase) (2) and total humidity (3) fields [16]

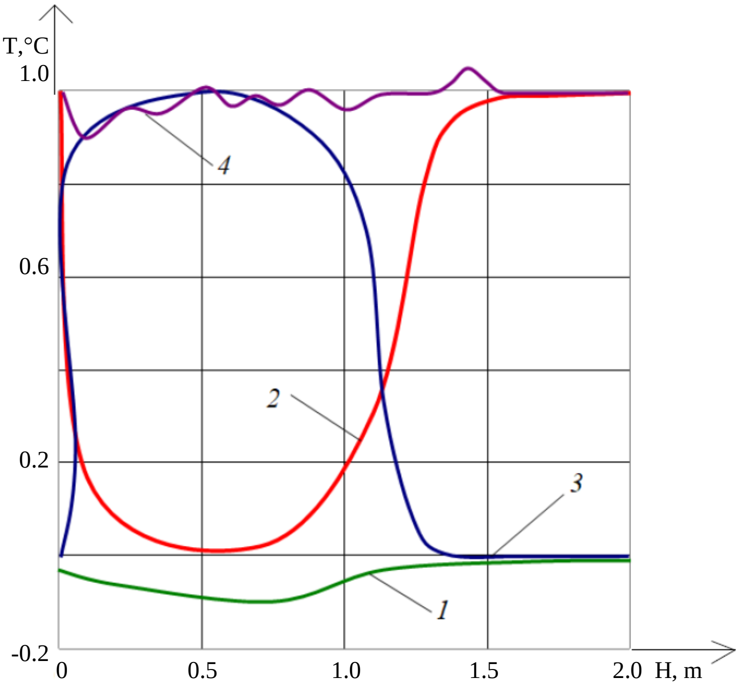

Figure 6 – Field distribution of temperature (1), moisture content (2) - liquid phase, (3) - solid phase - ice, concentration (4) [16]

Figure 6 shows that in the zone of active action of alternating temperatures (the rear front of ice formation) there is an inhomogeneous distribution of total concentration caused by alternating freezing and thawing of the ground. In addition, a significant inhomogeneity of the concentration distribution associated with displacement of the dissolved component is revealed at the front front of ice formation. This agrees with the data of numerical experiments conducted by Z.G. Ter-Martinosyan [15], V.I. Vasiliev (co-author) [16], I.I. Sakharov [17], and O.V. Tretyakova [18].

4. Conclusions

According to the results of the study it is reasonable to formulate the further outcomes:

- A thermohydrodynamic model including a system of three nonlinear equations (mass transfer equation, heat transfer equation and equilibrium equation) has been applied to study the nature of the effect of heat and mass transfer on pile foundations in seasonally freezing soils;
- The mathematical model of heat, moisture and salt transport in dispersed composition during freezing-thawing of soil is formulated taking into account the functional relation between the parameters determining the phase equilibrium, mutual influence of diffusion flows and convective transport;
- The character of heat and mass transfer indicates an increase in ice accumulation, which leads to inhomogeneous vat of seasonally freezing soil; taking into account these features have an important practical significance for predicting the stability and reliability of pile foundations
- As a result of numerical modelling it was determined that the distribution of temperature fields at a depth of 0.8 m varies from 0 to 0.1 $^{\circ}$c, the moisture content in the ground at temperatures up to 0 $^{\circ}$c is in the liquid phase, at lowering it turns into the solid phase.

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References

1. Foundations on seasonally freezing soils / A.L. Nevzorov. — Moscow: ASV, 2022. — 152 p.

2. SP RK 2.04-01-2017 Building climatology. — 2017.
3. Teoriya teplomassoperenosa. Modelirovanie granichnyh zadach: uchebnoe posobie dlya vuzov / A.I. Korotkij, Yu.V. Starodubceva. — Moscow: Izdatelstvo Yurajt, 2024. — 168 p.
4. Numerical modeling of thermal stabilization of filter ground / P.N. Vabishchevich, M.V. Vasilyeva, N.V. Pavlova // Matematicheskoe modelirovanie. — 2014. — Vol. 26, No. 6. — P. 111–125.
5. SP RK 5.01-103-2013 Pile foundations. — 2013.
6. Sposoby borby s moroznym pucheniem sezonnopromerzayushih gruntov v osnovaniyah fundamentov zdanij i sooruzhenij / D.S. Skvortsov, A.N. Kraev, A.N. Kraev, E.A. Zhaisambaev // The Eurasian Scientific Journal. — 2019. — Vol. 11, No. 5. — P. 1–12.
7. Teoriya teplomassoperenosa: reshenie zadach dlya mnogoslojnyh konstrukcij: uchebnoe posobie dlya vuzov / E.M. Kartashov, V.A. Kudinov, V.V. Kalashnikov. — Moscow: Izdatelstvo Yurajt, 2024. — 435 p.
8. Poromechanics / O. Coussy. — Chichester, UK: John Wiley & Sons, Ltd, 2003. — 298 p.
9. Chislennye metody resheniya zadach teplo-i massoperenosa / A.Yu. Krainov, L.L. Minkov. — Tomsk: STT, 2023. — 92 p.
10. Geotehnicheskoe modelirovanie processa promerzaniya i ottaivaniya morozoopasnyh gruntov / S.A. Kudryavtsev. — St. Petersburg: ASV, 2015. — 32 p.
11. SP RK 2.03-101-2012 Buildings and constructions in earned additionally territories and subsiding soil. — 2012.
12. Numerical solution of the problem of soil freezing / V.I. Popov, V.V. Popov // Mathematical modeling. — 2023. — Vol. 20, No. 5. — P. 121–130.
13. Deformacii i napryazheniya v promerzashih i ottaivayushih porodah / E.D. Ershov. — Moscow: MSU, 1985. — 167 p.
14. Metody granichnyh elementov v mehanike tverdogo tela / S. Crouch, A. Starfield. — Moscow: Mir, 1992. — 328 p.
15. Raschet kasatel'nogo napryazheniya moroznogo pucheniya vdol' stvola svai pri uchete yeye deformiruyemosti / Z.G. Ter-Martirosyan, P.A. Gorbachev // Zhilishchnoye stroitel'stvo. — 2022. — No. 12. — P. 36–39.
16. Mathematical Modeling of Temperature Regime of Soils of Foundation on Permafrost / V.I. Vasiliev, M.V. Vasilyeva, I.K. Sirditov, S.P. Stepanov, A.N. Tseev // Herald of the Bauman Moscow State Technical University. Series Natural Sciences. — 2017. — Vol. 70. — P. 142–159. https://doi.org/10.18698/1812-3368-2017-1-142-159
17. Reshenie trehmernoj temperaturno-vlazhnostnoj zadachi promerzaniya i pucheniya na primere maloetazhnogo kirpichnogo zdaniya / I.I. Sakharov, V.N. Paramonov, K.G. Shashkin // Razvitie gorodov i geotehnicheskoe stroitelstvo. — 2011. — No. 2. — P. 1–12.
18. Normalnye napryazheniya moroznogo pucheniya kak funkciya izbytochnoj vlazhnosti / O.V. Tretyakova // Magazine of Civil Engineering. — 2017. — Vol. 76, No. 8. — P. 130–139.

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