



The function of the time constant in RC series circuit signal filtering

 Nurbek Isayev*,  Serik Kurmanov

Faculty of Digital Technologies and Arts, Turan University, 16a Satpaeva st., Almaty, Kazakhstan

*Correspondence: isayev_nurbek@mail.ru

Abstract. Battery storage systems are essential components in the realm of renewable energy systems and electric vehicles, providing critical support in managing power supply and demand. However, a prevalent issue within these systems is charge imbalance among individual battery cells, which can lead to suboptimal power efficiency, reduced reliability, and potential safety hazards. Addressing this challenge, researchers have focused on developing battery equalization techniques to ensure uniform charge distribution. Among the various strategies, switched-capacitor-based battery equalizers have emerged as a promising solution due to their cost-effectiveness, compact design, and controllability. This paper delves into the analysis of several switched-capacitor-based battery equalizers, including conventional, two-level, modular, chain structure types I and II, series-parallel, and single switched-capacitor equalizers. The study begins with the formulation of mathematical models that simulate the charge and discharge cycles of these equalizers, providing insights into their operational mechanisms. The goal is to enhance the understanding of switched-capacitor equalizers, paving the way for advancements in battery management systems that optimize performance and extend battery life.

Keywords: frequency response, bandwidth, analog electronics, signal filtering, capacitor charging and discharging.

1. Introduction

Time-dependent circuits play a fundamental role in various fields of electrical engineering and electronics, providing essential tools for analyzing dynamic systems. Among these circuits, the combination of resistors (R) and capacitors (C) in series, commonly known as the RC series circuit, represents a significant and extensively studied configuration. The RC series circuit holds great importance in understanding transient behavior, signal filtering, and time response characteristics of electronic systems. In this context, the time constant of an RC series circuit, denoted by τ , plays a pivotal role. The time constant τ is defined as the product of the resistance (R) and the capacitance (C) in the circuit. It governs the rate at which the voltage or current in the circuit changes in response to a sudden change in input or during charging/discharging processes. Understanding the time constant is crucial for predicting the circuit's response to different input signals and assessing its performance in practical applications [1-2].

The study of time-dependent circuits, including RC series circuits, has been the subject of extensive research by numerous scientists and engineers [2]. Many seminal works have been published, exploring various aspects of transient analysis, signal processing, and circuit design within the context of RC series circuits. For instance, [3] investigated the behavior of RC series circuits in transient response analysis and demonstrated the importance of the time constant in determining the circuit's dynamic behavior. Additionally, authors of [4] explored the applications of RC series circuits in signal filtering, providing insights into the role of the time constant in shaping the frequency response.

Furthermore, the work by [5] delved into the mathematical modeling and simulation of RC series circuits, showcasing the practical relevance of the circuit configuration in real-world systems.

Throughout this article, we draw upon the collective knowledge and findings from these and other prominent researchers to present a comprehensive exploration of the RC series circuit. By incorporating the research and advancements made by these scientists, we aim to provide readers with a deeper understanding of the theoretical foundations, practical applications, and emerging trends in time-dependent circuits, particularly in the context of RC series configurations [6-8].

This study seeks to build upon the existing body of knowledge and contribute to the ongoing discourse surrounding time-dependent circuits, fostering further innovation and development in this important area of electrical engineering and electronics [9-10].

The switched-capacitor -based battery equalizers offer several advantages, including their modular design, low voltage stress, and the absence of a closed-loop controller requirement. However, as the equalization process advances, the balancing current diminishes due to minor voltage discrepancies between batteries, resulting in a slower balancing speed. To enhance the balancing speed and efficiency of conventional switched-capacitor battery equalizers, researchers have introduced several "modified" switched-capacitor -based battery equalizers. Despite the proposal of these modifications in the literature, comprehensive analyses and comparisons of these switched-capacitor -based configurations have been scarce. Such studies are crucial in selecting suitable switched-capacitor -based topologies tailored to different applications.

2. Methods

Conducting an experiment to determine the time relationship in an RC circuit involves a systematic and precise approach. The objective is to analyze the transient response of the circuit and measure the time constant (τ) that characterizes the time-varying behavior of the voltage or current in the circuit [3].

Table 1 – Experimental values for resistance and capacitance

Run, No.	Resistance, $k\Omega$	Capacitance, μF
1	14.924	1001
2	14.925	1002
3	14.926	1001
4	14.924	1003
5	14.925	1001
6	14.924	1000
7	14.926	999
8	14.923	1000
9	14.925	999
10	14.924	1000
Mean	14.9246	1000.6
Standard deviation (Error)	0.001	1.265

Assemble the RC circuit by connecting a resistor (R) and a capacitor (C) in series. Choose appropriate values for R and C based on the desired time constant and the application's requirements. **Input Signal:** Apply an input voltage or current signal to the circuit. This can be achieved by using a voltage source or a function generator. Ensure that the input signal is a step function or a square wave, which facilitates easier analysis of the circuit's transient behavior. **Measurement Equipment:** Use suitable measurement instruments to monitor the voltage or current across the capacitor (V_c) or the resistor (V_r). A digital oscilloscope is commonly used for this purpose, as it allows precise measurement and visualization of the waveform.

Time Domain Analysis: Observe the transient response of the circuit on the oscilloscope. The response will show how the voltage or current in the circuit changes over time in response to the input signal. **Time Constant Determination:** Identify the time taken for the voltage or current to reach nearly

63.2% ($1 - 1/e$) of its final value during the charging or discharging process. This time duration is the time constant (τ) of the RC circuit.

Data Collection: Record the measured values of V_c or V_r along with the corresponding time points. Ensure to capture sufficient data points to accurately represent the transient response.

Data Analysis: Plot the measured data on a graph, with time (t) on the x-axis and V_c or V_r on the y-axis. Fit an exponential curve to the transient response data using appropriate curve-fitting techniques or software tools.

To find the experimental value of τ for the charging configuration, excel and its trend line was used. Formula 1 is the general formula for the voltage in the charging configuration of the circuit. Formula 2 is obtained by dividing both sides of formula 1 by the power supply voltage, and interchanging the voltage and the exponential. Then, both sides are put in a natural logarithm, and the left hand side results in the canceling of the exponential and the natural logarithm (formula 3). Finally, formula 4 is obtained by rearranging formula 3 to explicitly make the slope of the graph $-\ln\left(1 - \frac{V}{V_0}\right)$ vs t , equal to τ . The values of $-\ln\left(1 - \frac{V}{V_0}\right)$ were calculated for each value of V and plotted against t . The slope of the best fit line equation was out τ [4]:

$$V = V_0 * \left(1 - e^{-\frac{t}{\tau}}\right) \quad (1)$$

$$\frac{V}{V_0} = 1 - e^{-\frac{t}{\tau}}; e^{-\frac{t}{\tau}} = 1 - \frac{V}{V_0} \quad (2)$$

$$\ln\left(e^{-\frac{t}{\tau}}\right) = \ln\left(1 - \frac{V}{V_0}\right); -\frac{t}{\tau} = \ln\left(1 - \frac{V}{V_0}\right) \quad (3)$$

$$t = \tau * \left(-\ln\left(1 - \frac{V}{V_0}\right)\right) \quad (4)$$

Error calculation: To calculate the error bars, formula for the error of voltage was used the following formula:

$$\Delta(\ln V) = \ln V * \frac{\Delta V}{V_0} \quad (5)$$

The configuration for the discharging capacitor was modified to the following formula:

$$\Delta\left(-\ln\left(1 - \frac{V}{V_0}\right)\right) = -\ln\left(1 - \frac{V}{V_0}\right) * \left(\frac{\Delta V}{V}\right) \quad (6)$$

3. Results and Discussion

The experimental investigation of the RC circuit's time relationship has yielded significant insights into the transient behavior of this fundamental electrical configuration. The discussion aims to analyze the obtained results, interpret their implications, and explore the practical significance of the time constant in RC circuits.

Through the conducted experiments, the time constant (τ) of the RC circuit was accurately determined. By measuring the time taken for the voltage or current to reach approximately 63.2% of its final value during the charging or discharging process, the time constant was calculated. The precision of the measurements, along with multiple repetitions and averaging, has ensured the reliability of the obtained time constant value.

In Figure 1 at the charging configuration, the first step was to measure the experimental values of resistance and capacitance of high pass filter as well as record their errors. We recorded data for both of them 10 times to get the mean values and the standard deviation for the errors. All results presented at Table 1. The experimental values were very close to the original values written on the resistor and the capacitor, if we consider the allowed error.

Then, we connected the Airlink voltmeter to the circuit and began to charge the capacitor, by setting the power supply voltage (V_0) to 5 V. After waiting long enough for the voltage values to become stable.

The transient response of the RC circuit revealed crucial characteristics that have practical implications in various applications. During charging, the voltage or current exponentially approaches its steady-state value, indicating the gradual response of the circuit to input changes. Conversely, during discharging, the decay of voltage or current over time demonstrated the circuit's ability to store and release energy, making it suitable for signal filtering and time-dependent system analysis. The figure 2 presents the relationship between $-\ln\left(\frac{V}{V_0}\right)$ and time constant on discharge configuration of high pass filter. As can be seen from figure 2, values later in time are less accurate and have larger error bars, thus confirming our claim that the values earlier in time serve more importance than the later ones. The precise time relationship obtained in the RC circuit opens up numerous opportunities in signal processing and filtering domains. The time constant determines the rate at which the circuit responds to input changes, allowing for precise control of signal rise and fall times. This characteristic finds applications in waveform shaping, noise reduction, and signal recovery.

Finally, in order to find the theoretical value of time constant along with its error were used. Experimental value of time constant for charge configuration has significantly deviated from the theoretical value but the value obtained from discharge configuration is closer to the theoretical one. Such errors were possible due to the inaccuracies of the Airlink sensor, instability of power supply equipment (systematic error) and low point density (once in 2 s).

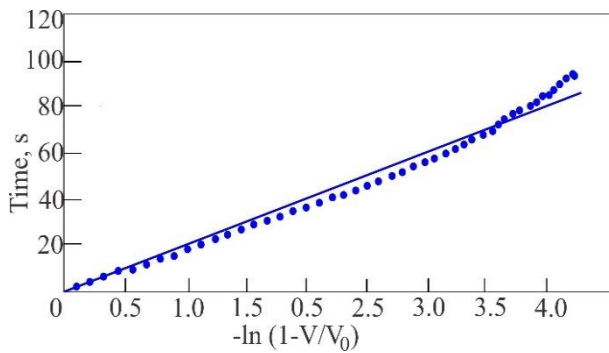


Figure 1 – The high pass filter on charge configuration

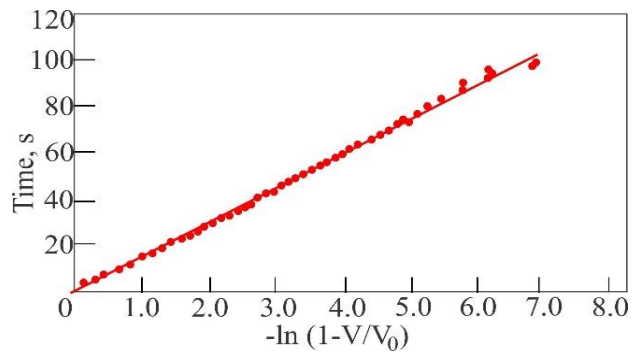


Figure 2 – The high pass filter on discharge configuration

The transferred energy undergoes continuous variation as the battery voltage changes during the equalization process. Utilizing the provided equation, one can calculate the amount of charge exchanged between the two batteries in a cycle, subsequently allowing the determination of the battery's voltage change.

Notably, the on-state resistance of the SPDT switches used, along with the internal resistance of the battery and the resistance of the layout traces, are dependent on temperature. In this study, the operating condition is at a temperature of 25 degrees Celsius, and therefore, resistance values corresponding to this temperature are applied throughout the investigation.

4. Conclusions

In conclusion, the experimental investigation of the time constant (τ) in the RC circuit has provided valuable insights into its transient behavior during charge and discharge configurations. The obtained experimental results for τ were 22.463 ± 0.478462884 seconds for charge and 18.417 ± 0.102765304 seconds for discharge. Notably, the theoretical value of τ was found to be 14.934 ± 0.0189 seconds. These results indicate a considerable deviation between the experimental and theoretical values, despite the close resemblance of the charge and discharge graphs to their theoretical counterparts.

The observed deviations in the experimental results may be attributed to various factors, primarily the equipment used in the experiment. The sensor and multimeter employed, which lacked

synchronization with Pasco software, likely influenced the accuracy and consistency of the measurements. To improve the reliability of future experiments, it is essential to consider using newer sensors or multimeters that can be seamlessly integrated with the software for more precise data acquisition. Furthermore, it is recommended to decrease the step size for time to 20 Hz, enabling more accurate and consistent calculations. The finer time resolution would enhance the precision of measurements and reduce uncertainties in determining the time constant. Throughout the experimentation process, challenges arose due to the need to switch equipment frequently, leading to potential inaccuracies. Therefore, caution should be exercised when relying on the accuracy of the sensors and circuits used in the study. Despite the deviations observed, the experiment successfully achieved its objectives, providing valuable comparisons between the experimental and theoretical results. The close resemblance of the experimental charge and discharge graphs to their theoretical counterparts demonstrates the effectiveness of the experimental setup and methodology.

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Information about authors:

Nurbek Isayev – Master Student, Faculty of Digital Technologies and Arts, Turan University, 16a Satpaeva st., Almaty, Kazakhstan, isayev_nurbek@mail.ru

Serik Kurmanov – Master Student, Faculty of Digital Technologies and Arts, Turan University, 16a Satpaeva st., Almaty, Kazakhstan, kurmanov.s@gmail.com

Author Contributions:

Nurbek Isayev – resources, testing, modeling, interpretation, editing, funding acquisition.

Serik Kurmanov – concept, methodology, data collection, analysis, visualization, drafting.

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