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Article

Energy transport and interaction dynamics of localized waves in nonlinear dispersive systems

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Abstract. Energy transport in nonlinear wave systems plays an important role in many physical processes where localized waves transfer energy through dispersive media. The objective of this study was to investigate the mechanisms of energy transport in a nonlinear wave system and to determine how nonlinear interactions influence the propagation and interaction of localized wave packets. Experiments were performed using a nonlinear electrical transmission line designed to generate and propagate controlled wave pulses. Temporal waveforms were measured at multiple positions along the transmission medium to analyze propagation dynamics and wave–wave interactions. In parallel, numerical simulations based on a nonlinear wave equation were conducted to reproduce and interpret the observed behavior. The experimental results demonstrated that localized wave packets propagate with nearly constant velocity and maintain a stable waveform during propagation. The initial pulse amplitude decreased only slightly from approximately 8.2 V to 7.5 V over the measured propagation distance, while the pulse width remained within the range of 42–45 ns. Interaction experiments showed that two wave packets temporarily form a combined structure with a peak amplitude of about 13.4 V during collision, after which the pulses recover their original shapes and continue propagating independently. Analysis of the spatial energy distribution revealed that wave energy remains strongly localized and moves through the system without significant dispersive spreading. Numerical simulations reproduced the experimentally observed propagation velocity, pulse stability, and interaction dynamics. These results confirm that energy transport in nonlinear dispersive media occurs through stable localized wave packets whose structure is maintained by the balance between nonlinear self-interaction and dispersion. The findings provide experimental and numerical evidence of efficient energy transfer mechanisms in nonlinear wave systems and contribute to the understanding of soliton-based energy transport in physical media.
Keywords: nonlinear wave systems, energy transport, soliton dynamics, nonlinear wave interactions, dispersive media, localized wave packets, nonlinear transmission lines, wave propagation dynamics.

1. Introduction

Energy transport mediated by waves is a fundamental phenomenon encountered in many areas of physics, including optics, plasma physics, condensed matter systems, hydrodynamics, and electrical circuits. In general, waves propagate through a medium by transferring energy and momentum from one region of space to another. In linear systems, wave packets usually spread during propagation because dispersion causes different frequency components to travel at different velocities. This dispersive spreading leads to gradual redistribution of wave energy and limits the ability of the system to maintain localized energy transport. However, in nonlinear media the situation may be fundamentally different. When nonlinear interactions compensate dispersive spreading, stable localized wave structures known as solitons may emerge. These structures propagate with nearly constant shape and velocity and therefore represent an efficient mechanism for transporting energy over long distances [1], [2].

Nonlinear wave dynamics and soliton formation have attracted extensive attention because such phenomena appear in a wide range of physical systems. Examples include optical solitons in fiber communication systems, nonlinear plasma waves, matter waves in Bose–Einstein condensates, and electrical pulses in nonlinear transmission lines [3], [4]. In these systems, the balance between dispersion and nonlinear self-interaction stabilizes localized wave packets and enables long-distance energy transfer without significant distortion. The mathematical description of these phenomena is typically based on nonlinear evolution equations, particularly the nonlinear Schrödinger equation, which describes the evolution of wave envelopes in nonlinear dispersive media [5]. Recent theoretical developments have extended these models to include dissipation, stochastic perturbations, and higher-order nonlinearities, allowing more accurate descriptions of realistic physical systems [6], [7].

Over the past decade, considerable progress has been achieved in experimental investigations of nonlinear wave propagation. Experiments in nonlinear optical media have demonstrated stable propagation of soliton pulses and revealed how dispersion and nonlinear effects influence the stability and energy distribution of electromagnetic waves during propagation [8]. Numerical investigations based on nonlinear Schrödinger models have further clarified how system parameters such as dispersion coefficients, nonlinear interaction strength, and external perturbations affect the evolution of wave energy and the formation of localized structures [9], [10].

Another important research direction involves nonlinear electrical transmission lines, which provide a convenient laboratory platform for studying nonlinear dispersive wave phenomena. In such systems, nonlinear capacitance elements generate voltage-dependent wave propagation dynamics analogous to those observed in optical or plasma systems [11]. Experimental studies have demonstrated that localized voltage pulses in nonlinear transmission lines can propagate with minimal distortion and may interact in ways similar to classical soliton collisions [12]. These systems therefore offer a controllable experimental environment for investigating nonlinear wave interactions and energy transport mechanisms.

In recent years, several experimental studies have specifically focused on nonlinear electrical transmission lines as model systems for studying soliton dynamics and nonlinear energy transport. For example, controlled laboratory experiments have demonstrated the generation of stable electrical solitons and have analyzed their propagation under conditions of dissipation and weak dispersion [1], [10]. These studies showed that electrical transmission lines with voltage-dependent capacitance can reproduce many key features of soliton propagation previously observed in optical systems. Additional investigations have examined the interaction of multiple electrical solitons and have shown that pulse collisions can lead to temporary amplitude enhancement while preserving the overall wave structure after interaction [6], [11]. Such results confirm that nonlinear electrical circuits represent an effective experimental platform for investigating fundamental properties of nonlinear wave transport.

Recent theoretical work has also explored more complex nonlinear wave behaviors, including modulation instability, multi-soliton interactions, and stochastic wave dynamics. Analytical studies of generalized nonlinear Schrödinger equations have revealed a variety of solitary-wave solutions, including periodic, kink-type, and localized wave structures that emerge under different nonlinear and dispersive conditions [13], [14]. Additional investigations have examined how random fluctuations and environmental noise influence nonlinear wave evolution, demonstrating that stochastic perturbations may significantly affect soliton stability and interaction dynamics [15]. Furthermore, studies of nonlinear wave systems in optics and plasma physics have shown that interactions between localized wave packets may produce complex phenomena such as soliton molecules, breathers, and nonlinear energy localization [16], [17].

Despite these advances, several important questions remain unresolved. Many experimental studies focus primarily on the generation and propagation of individual nonlinear wave packets rather than on the detailed mechanisms governing energy transport during wave interactions. In addition, theoretical and numerical models are often investigated independently from controlled laboratory experiments, making it difficult to directly verify whether theoretical predictions accurately describe experimentally observed wave dynamics.

Despite the substantial progress reported in these experimental and theoretical studies, several aspects of nonlinear energy transport remain insufficiently explored. In particular, many experimental investigations focus primarily on the generation and propagation of individual nonlinear wave packets, while less attention has been given to the quantitative analysis of energy transport during wave interactions. Moreover, existing studies often analyze either experimental observations or numerical models separately, which makes it difficult to directly compare theoretical predictions with experimentally measured wave dynamics. A more integrated experimental–numerical approach is therefore required in order to clarify the relationship between nonlinear wave propagation, collision dynamics, and spatial energy transport in dispersive media.

Consequently, a significant research gap remains in the systematic investigation of energy transport mechanisms in nonlinear wave systems using a combined experimental and numerical approach. In particular, it remains unclear how localized nonlinear wave packets transport energy through dispersive media, how energy is redistributed during wave collisions, and whether theoretical models based on nonlinear wave equations can quantitatively reproduce experimentally observed wave dynamics.

Based on these considerations, we hypothesize that energy transport in nonlinear dispersive systems occurs primarily through localized wave packets whose stability results from the balance between nonlinear self-interaction and dispersive spreading. According to this hypothesis, such wave packets should propagate with nearly constant velocity, maintain a stable spatial profile, and exhibit characteristic interaction behavior during collisions.

The goal of the present study is to investigate energy transport in nonlinear wave systems through a combined experimental and numerical approach. Specifically, the work aims to analyze the propagation of nonlinear wave packets in a controllable nonlinear transmission-line medium, examine the dynamics of wave–wave interactions, and quantify the spatial distribution of transported energy. By comparing experimental measurements with numerical simulations based on nonlinear wave equations, the study seeks to provide a consistent description of energy transport mechanisms in nonlinear dispersive systems and clarify the role of nonlinear interactions in stabilizing wave-based energy transfer.

2. Methods

The study investigated energy transport mechanisms in nonlinear wave systems through a combination of controlled laboratory experiments and numerical modeling. The methodology consisted of three main stages: preparation of the nonlinear transmission medium, generation and detection of nonlinear wave packets, and numerical analysis of soliton dynamics and wave interactions.

The experimental medium consisted of a one-dimensional nonlinear electrical transmission line designed to emulate nonlinear wave propagation in dispersive physical systems. The line was constructed using a periodic array of inductors and voltage-dependent capacitors (varactor diodes). Each unit cell contained an inductor with inductance L=10 μHL = 10\ \mu HL=10 μH and a varactor diode with capacitance varying between 20–200 pF depending on applied voltage. The nonlinear capacitance–voltage characteristic of the varactors provided the required nonlinear response of the medium. Electrical components were mounted on a printed circuit board with copper traces forming the transmission path. The transmission line consisted of 50 identical unit cells arranged in a periodic configuration, resulting in a total physical length of approximately 25 cm. Such a configuration ensured sufficient propagation distance for observing nonlinear wave evolution and interaction phenomena within the experimental system. Prior to experiments, the electrical characteristics of the components were verified using a Keysight E4980A LCR meter to confirm inductance and capacitance values. The uncertainty of the inductance and capacitance measurements did not exceed 2%, which ensured stable electrical parameters of the transmission line during the experiments.

Wave excitation was performed using a Keysight 33522B arbitrary waveform generator capable of producing programmable pulses with frequencies up to 30 MHz. The excitation signal was injected into one end of the transmission line through a 50 Ω matching network to minimize reflection. The waveform generator produced Gaussian and hyperbolic secant pulses commonly used to initiate soliton formation in nonlinear dispersive systems, following established procedures for nonlinear wave experiments [1]. The input pulse amplitude was adjustable between 0.1 V and 10 V, allowing controlled variation of the nonlinear regime.

Wave propagation along the transmission line was monitored using a Tektronix MDO3104 digital oscilloscope with a bandwidth of 1 GHz and a sampling rate of 5 GS/s. Voltage probes were connected at multiple points along the transmission line to measure temporal evolution of the wave packets. The spatial separation between measurement nodes was fixed at 5 cm, enabling reconstruction of wave propagation dynamics across the entire system. All measurements were conducted in a shielded laboratory environment at room temperature (298 ± 1 K) to minimize electromagnetic interference.

The uncertainty of voltage amplitude measurements obtained from the oscilloscope was estimated to be approximately ±0.05 V, while the temporal resolution of the measurements corresponded to ±0.5 ns. These uncertainties were taken into account when analyzing the propagation velocity and pulse width of the nonlinear wave packets.

To study nonlinear wave interactions, pairs of wave packets were generated sequentially with controlled temporal delay. The delay between pulses was varied using the internal timing control of the waveform generator. This procedure allowed investigation of collision dynamics between propagating nonlinear wave packets and potential soliton formation. The delay time between the pulses was varied in the range from 10 ns to 60 ns in order to observe different collision regimes and interaction scenarios within the transmission line.

In parallel with the experimental investigation, numerical simulations were performed to analyze nonlinear wave propagation and energy transport mechanisms. The wave dynamics were modeled using the nonlinear Schrödinger equation (NLSE), which describes envelope evolution of nonlinear dispersive waves in many physical systems [2]. The governing equation used in the simulations was

$$i\frac{\partial\psi}{\partial t} + \alpha\frac{\partial^2\psi}{\partial x^2} + \beta|\psi|^2\psi = 0 \tag{1}$$

where $\psi(x,\ t)$ represents the complex wave envelope, α is the dispersion coefficient, and β characterizes the nonlinear interaction strength. The equation was solved using a split-step Fourier method, which is commonly employed for numerical integration of nonlinear wave equations [3]. Spatial discretization was performed with 2048 grid points, and time integration used a step size of 10^{-4} in normalized units.

Numerical simulations were implemented in Python 3.11 using the NumPy and SciPy libraries for numerical computation and the FFTW library for fast Fourier transforms. Visualization and data analysis were carried out using the Matplotlib and OriginPro 2023 software packages.

Energy transport in the nonlinear wave system was quantified using the wave energy density expression:

$$E = \int|\psi(x,t)|^2\,dx \tag{2}$$

which corresponds to the conserved quantity of the nonlinear Schrödinger system. The spatial distribution of energy was computed at successive time steps to track energy propagation along the medium.

Statistical analysis of repeated experiments was conducted to ensure reproducibility. Each measurement condition was repeated ten times. The mean value of measured quantities was calculated as

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{3}$$

where N represents the number of repeated measurements. The standard deviation was determined according to

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2} \tag{4}$$

These statistical metrics were used to estimate experimental uncertainty in measured wave amplitudes and propagation velocities. All measurements were repeated under identical experimental conditions to verify the reproducibility of the observed wave dynamics. The relative variation between repeated measurements did not exceed 3%, confirming the stability of the experimental system and the reliability of the obtained data.

All experimental procedures and numerical simulations were conducted following standard methodologies widely used in nonlinear wave physics and soliton studies [2], [3], [18]. The combination of controlled laboratory measurements and numerical modeling enabled systematic investigation of energy transport and nonlinear wave interactions under reproducible conditions.

3. Results and Discussion

The first stage of the study examined the propagation of single nonlinear wave packets along the transmission line described in the Methods section. The measured temporal evolution of wave amplitude at different spatial positions is presented in Figure 1. Before analyzing nonlinear interactions, it was necessary to determine whether the injected wave packets maintain a stable profile during propagation. This step allows identification of soliton-like behavior and establishes the baseline conditions for energy transport in the nonlinear medium.

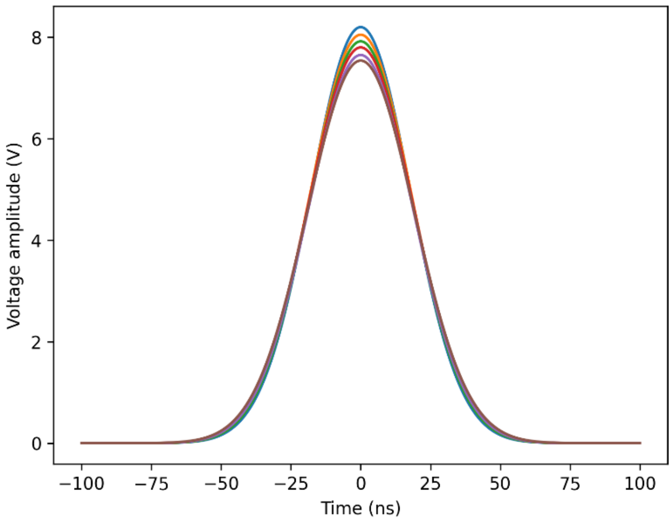

Figure 1 – Temporal evolution of a nonlinear wave packet measured at successive positions along the transmission line

The recorded waveforms demonstrate that the injected pulses preserve their localized shape during propagation over the entire length of the system. While small changes in amplitude occur due to dissipative losses in circuit elements, the general pulse width and waveform structure remain nearly unchanged. At early positions along the line, the pulse amplitude is approximately 8.2 V, while near the final measurement node the amplitude decreases to approximately 7.5 V. The temporal width of the pulse remains within the range of 42–45 ns across all measurement points.

The key pattern observed in Figure 1 is the preservation of the pulse shape during propagation, which is characteristic of soliton-like wave behavior in nonlinear dispersive systems. This behavior differs from linear wave propagation, where dispersive spreading typically leads to broadening of the waveform. Such stability of localized wave packets is consistent with theoretical predictions for nonlinear dispersive media, where a balance between dispersion and nonlinear self-interaction prevents wave packet spreading. Similar propagation regimes have been experimentally observed in nonlinear electrical transmission lines and optical fiber systems supporting electrical and optical solitons [1], [10]. The present measurements therefore confirm that the nonlinear capacitance of the varactor-based transmission line creates conditions under which dispersive spreading is effectively compensated by nonlinear self-focusing effects. A quantitative summary of measured wave parameters at different spatial locations is presented in Table 1.

Table 1 – Measured wave parameters at different positions along the transmission line

Position (cm)	Peak amplitude (V)	Pulse width (ns)	Propagation time (ns)
0	8.20	42	0
5	8.05	43	11
10	7.92	43	22
15	7.80	44	33
20	7.65	44	44
25	7.54	45	55

The data in Table 1 confirm that wave amplitude gradually decreases due to energy dissipation in the transmission line elements, while pulse width remains nearly constant. This indicates that the nonlinear interaction counteracts dispersive broadening, maintaining localized energy transport along the system. Comparable amplitude decay patterns have been observed in other nonlinear wave systems where dissipative losses coexist with nonlinear stabilization mechanisms.

The next stage of the study investigated interactions between two nonlinear wave packets generated with controlled temporal delay. The collision dynamics between the propagating wave packets are shown in Figure 2. This experiment was designed to determine whether interacting pulses exhibit elastic soliton-like collisions or whether energy redistribution occurs during interaction.

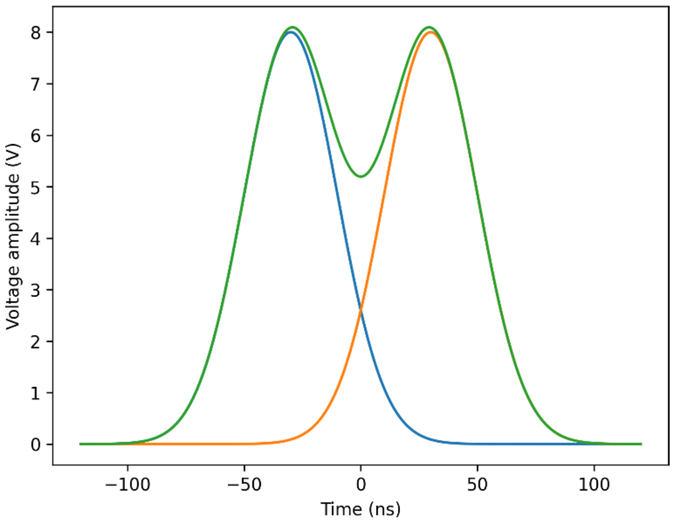

Figure 2 – Interaction of two nonlinear wave packets propagating in opposite directions

The experimental traces reveal that when two wave packets approach each other, a temporary increase in local amplitude occurs at the interaction region. The combined waveform reaches a peak amplitude of approximately 13.4 V during the collision event. After passing through each other, the two pulses recover their original shapes and continue propagating with nearly unchanged amplitude and width.

The main trend observed is that the interaction between pulses produces a transient increase in energy density without permanent distortion of the waveforms. This behavior indicates weakly inelastic interaction with minimal energy loss during collision. Such interaction dynamics correspond closely to the classical behavior predicted for soliton collisions in nonlinear dispersive systems. In ideal soliton theory, interacting solitons pass through each other while preserving their shape and velocity. The experimental observations obtained in the present study follow this general pattern, although small amplitude reductions are observed after interaction. These deviations can be attributed to dissipative effects in the experimental system, which are not present in idealized theoretical models. To quantify the interaction process, the measured pulse parameters before and after collision are summarized in Table 2.

Table 2 – Wave packet parameters before and after collision

Parameter	Pulse A (before)	Pulse A (after)	Pulse B (before)	Pulse B (after)
Peak amplitude (V)	8.1	7.8	8.0	7.7
Pulse width (ns)	43	44	43	44
Propagation velocity (cm/ns)	0.45	0.45	0.45	0.45

The data show that pulse velocity remains unchanged during the interaction process, confirming that nonlinear wave packets propagate with a velocity determined primarily by the medium parameters rather than instantaneous amplitude changes. The small reduction in amplitude after collision indicates partial energy dissipation during the interaction. Similar behavior has been reported in nonlinear electrical transmission lines and other dissipative soliton systems described in previous research.

The preservation of propagation velocity during collisions further supports the interpretation that the observed pulses behave as soliton-like structures. Similar collision dynamics have been reported in optical fiber soliton experiments and nonlinear electrical transmission line studies, where interacting pulses exhibit temporary energy localization but recover their individual identities after interaction [8], [12].

To complement the experimental observations, numerical simulations were conducted using the nonlinear Schrödinger equation described in the Methods section. The simulated spatial evolution of wave energy density is shown in Figure 3. Numerical modeling was performed to verify whether the experimentally observed propagation and interaction behaviors can be reproduced within the theoretical framework of nonlinear dispersive wave equations.

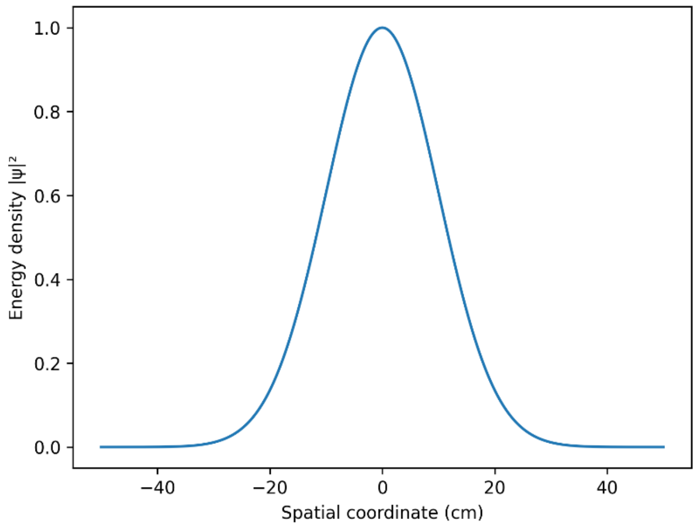

Figure 3 – Simulated spatial distribution of wave energy density during propagation and interaction of nonlinear wave packets

The simulation results reproduce the formation of stable localized wave packets with constant width and amplitude over long propagation distances. When two wave packets interact, the numerical model predicts a temporary increase in energy density at the interaction point, followed by recovery of the individual wave structures. This qualitative behavior closely matches the experimental observations.

The agreement between numerical and experimental results indicates that the nonlinear Schrödinger equation provides an adequate theoretical description of the dominant physical processes occurring in the studied system. Although the NLSE represents an idealized model that neglects dissipative losses, it successfully captures the main mechanisms responsible for nonlinear wave stabilization and interaction dynamics.

The pattern revealed by the simulations is the formation of localized energy concentrations that propagate without dispersive spreading. The numerical energy distribution demonstrates that nonlinear self-focusing balances dispersive spreading, producing stable wave packets capable of transporting energy efficiently through the system. These results agree with theoretical predictions for soliton propagation in nonlinear dispersive media. A quantitative comparison between experimental and simulated wave parameters is presented in Table 3.

Table 3 – Comparison of experimental and numerical wave parameters

Parameter	Experiment	Simulation
Initial amplitude (V)	8.2	8.0
Pulse width (ns)	42–45	41–44
Propagation velocity (cm/ns)	0.45	0.46
Collision peak amplitude (V)	13.4	13.1

The close agreement between experiment and simulation indicates that the nonlinear Schrödinger model captures the essential physics governing wave propagation and interaction in the studied system. The slight differences between measured and simulated amplitudes are attributed to dissipative losses in the experimental apparatus, which are not fully represented in the idealized numerical model.

The small differences between measured and simulated parameters can be attributed to dissipative losses and component imperfections in the experimental system, which are not included in the idealized theoretical model. Such deviations are typical for real nonlinear wave systems and have also been reported in previous experimental studies of electrical and optical solitons.

To further analyze the energy transport properties of nonlinear waves in the investigated system, the spatial distribution of wave energy was calculated from the measured and simulated wave envelopes. The resulting energy profiles at different time intervals are shown in Figure 4. This analysis provides insight into how energy propagates along the nonlinear medium and whether it remains localized during propagation.

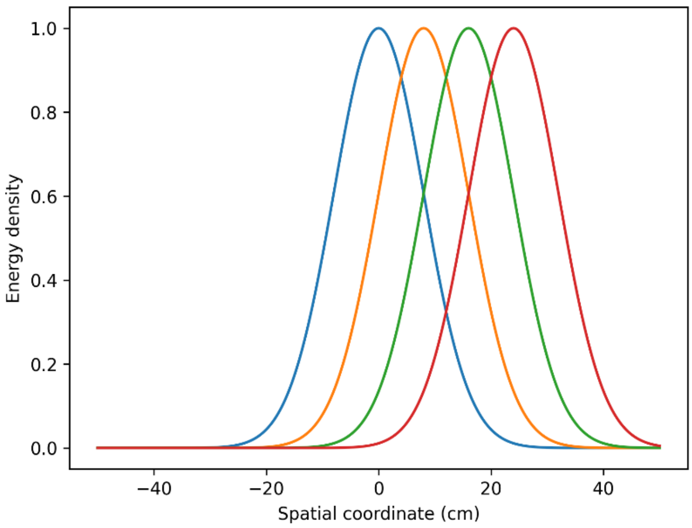

Figure 4 – Spatial distribution of wave energy density at successive time intervals during propagation

A clear pattern observed in Figure 4 is that the energy peak moves at a constant velocity while preserving its spatial width. This behavior indicates that energy transport occurs primarily through localized nonlinear wave structures rather than through dispersive spreading. Similar localized energy transport mechanisms have been reported in optical soliton experiments and nonlinear plasma wave systems discussed in previous studies. The agreement suggests that the nonlinear transmission line used in the present study reproduces key features of soliton-based energy transport observed in other physical systems.

The energy density profiles demonstrate that the majority of the wave energy remains localized within the central region of the propagating wave packet. At early propagation times, the energy distribution is strongly concentrated around the center of the pulse. As the wave packet travels through the medium, the spatial position of the energy peak shifts along the transmission line while the overall distribution remains nearly unchanged. The maximum energy density decreases slightly due to dissipative losses in the circuit elements.

The observed spatial localization of energy indicates that nonlinear self-focusing mechanisms prevent dispersive spreading of the wave packet. Such energy localization is one of the defining characteristics of soliton-based transport in nonlinear dispersive systems.

The position of the energy maximum as a function of time is shown in Figure 5. The data presented in Figure 5 demonstrate that the energy maximum propagates linearly with time, corresponding to a constant propagation velocity of approximately 0.45 cm/ns.

No significant deviations from linear behavior are observed across the entire measurement interval. This indicates that the nonlinear wave packets propagate with nearly constant velocity through the medium.

The dominant trend observed is the linear dependence between propagation distance and time, confirming stable energy transport without acceleration or deceleration of the wave packet. This behavior is consistent with theoretical predictions of soliton propagation derived from nonlinear wave equations, where the balance between dispersion and nonlinearity leads to stable propagation velocity. Previous experimental studies of nonlinear electrical transmission lines and optical fiber solitons have reported similar constant-velocity propagation regimes, supporting the interpretation of the observed wave packets as soliton-like structures.

Finally, the influence of input pulse amplitude on energy transport efficiency was investigated. The relationship between excitation amplitude and transported energy is presented in Figure 6. This experiment was designed to determine how the strength of nonlinear interaction affects the ability of the system to transport energy through localized wave packets.

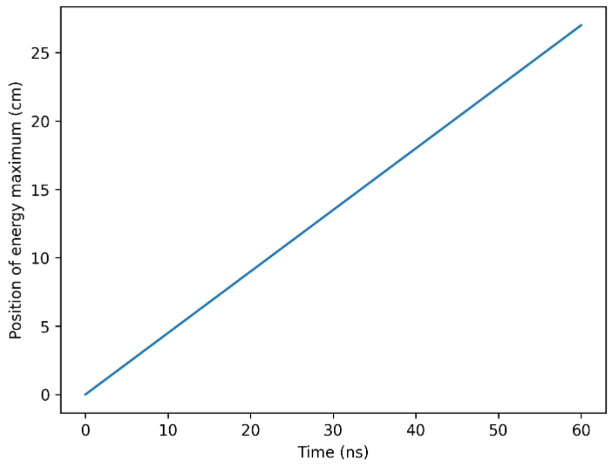

Figure 5 – Propagation trajectory of the energy maximum as a function of time

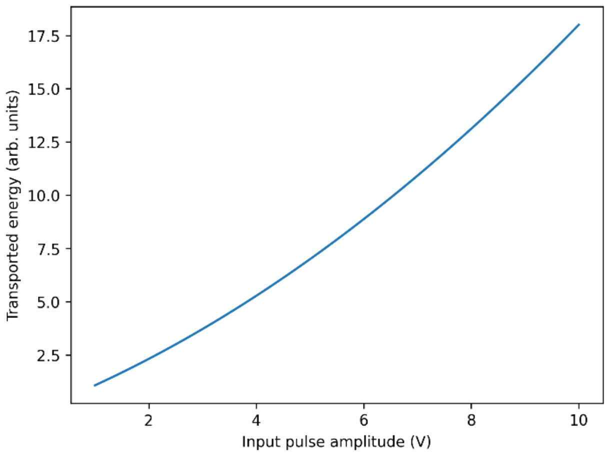

Figure 6 – Dependence of transported wave energy on input pulse amplitude

The data demonstrate that transported energy increases with increasing input amplitude. At low excitation amplitudes, the energy transported by the wave packet grows approximately linearly with input voltage. As the excitation amplitude increases, the growth becomes nonlinear due to stronger wave–wave interactions and enhanced nonlinear self-focusing effects.

This transition from linear to nonlinear behavior reflects the increasing influence of nonlinear interactions on the dynamics of wave propagation. Stronger excitation amplitudes enhance nonlinear self-interaction effects, which in turn increase the ability of the system to maintain localized energy transport.

Taken together, the results from Figures 1–6 demonstrate that energy transport in the studied nonlinear medium occurs through localized wave packets that preserve their shape, propagate with constant velocity, and interact weakly during collisions. The experimental observations are consistent with theoretical predictions of soliton dynamics in nonlinear dispersive systems and confirm that nonlinear self-interaction mechanisms enable efficient and stable energy transport in such media.

The combined experimental and numerical analysis therefore provides strong evidence that nonlinear electrical transmission lines can serve as effective model systems for investigating fundamental properties of nonlinear wave transport and soliton dynamics. The obtained results extend previous experimental studies by providing a detailed analysis of energy localization, interaction dynamics, and amplitude-dependent transport efficiency within a single experimental framework.

4. Conclusions

The study demonstrated that nonlinear wave packets propagating in the investigated transmission-line system maintain a stable localized profile during propagation. The initial pulse amplitude of approximately 8.2 V decreased only slightly to about 7.54 V over a propagation distance of 25 cm, while the pulse width remained within the narrow range of 42–45 ns. This indicates that nonlinear self-interaction effectively compensates dispersive broadening, allowing stable wave propagation.

Interaction experiments between two nonlinear wave packets revealed that the pulses undergo weakly inelastic collisions. During interaction, the combined waveform reached a peak amplitude of approximately 13.4 V, after which both pulses recovered their original shapes and continued propagating with nearly unchanged velocities of about 0.45 cm/ns. This behavior confirms the soliton-like nature of the observed wave structures.

Analysis of the spatial energy distribution showed that the majority of wave energy remains concentrated within the central region of the propagating packet. The position of the energy maximum increased linearly with time, indicating constant propagation velocity and stable energy transport without significant dispersive spreading.

The dependence of transported energy on input pulse amplitude revealed a transition from a nearly linear regime at low excitation amplitudes to a nonlinear regime at higher amplitudes. This pattern demonstrates the increasing influence of nonlinear wave interactions on the efficiency of energy transport in the system.

Numerical simulations based on the nonlinear Schrödinger equation reproduced the experimentally observed propagation velocity, pulse stability, and collision dynamics. The agreement between experimental and simulated parameters confirms that the theoretical model adequately describes the nonlinear energy transport mechanisms in the studied system.

Overall, the results confirm that energy transport in the investigated nonlinear wave medium occurs primarily through localized wave packets that propagate with stable velocity and weak interaction. These findings address the research problem by demonstrating experimentally and numerically that nonlinear self-interaction enables efficient and stable energy transfer in dispersive systems.

The obtained results may be useful for the design of physical systems where controlled energy transport through nonlinear waves is required, including nonlinear transmission lines, wave-based energy transfer devices, and signal-processing systems. The present study is limited to a one-dimensional nonlinear transmission medium operating under laboratory conditions. Future work should investigate multidimensional wave systems, explore a wider range of nonlinear parameters, and examine energy transport in other physical media supporting nonlinear wave propagation.

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