



Numerical framework for simulating quantum spacetime fluctuations with prescribed spectra

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Abstract. Understanding quantum fluctuations of spacetime at the Planck scale remains one of the central challenges of theoretical physics. This study introduces a stochastic framework to model such fluctuations, aiming to test whether a phenomenological approach can reproduce expected statistical signatures of quantum geometry. The metric field was represented as a one-dimensional Gaussian process with a prescribed power spectrum, and its Fourier modes were evolved through an Ornstein–Uhlenbeck process to enforce stationarity. Numerical simulations were carried out on a discretized domain with periodic boundary conditions, and statistical analyses were performed on power spectra, spatial correlations, temporal autocorrelations, and field distributions. The results showed that the empirical power spectrum reproduced the target distribution across more than two decades in wavenumber, with a clear suppression of high-frequency modes due to ultraviolet damping. The spatial correlation function indicated a coherence scale of approximately 150–200 Planck lengths, beyond which fluctuations decorrelate, making spacetime effectively smooth at larger scales. Temporal autocorrelations decayed exponentially with a relaxation time of about 200 Planck units, demonstrating that spacetime fluctuations possess finite memory. The field amplitudes followed a Gaussian distribution, supporting the assumption of central-limit behavior in the linear regime. Stationary field snapshots confirmed equilibrium behavior throughout the simulation. Overall, the study establishes a reproducible and computationally efficient framework for simulating Planck-scale metric fluctuations. The findings highlight short correlation lengths, finite coherence times, and Gaussian statistics as key features of quantum spacetime, providing a bridge between phenomenological modeling and fundamental theory..

Keywords: quantum spacetime fluctuations, stochastic modeling, power spectrum, spatial correlation, temporal autocorrelation, Gaussian statistics, Planck-scale dynamics.

1. Introduction

The nature of spacetime at the Planck scale has long been regarded as one of the deepest unsolved problems in fundamental physics [1]. While general relativity treats spacetime as a smooth continuum, quantum theory predicts that it must fluctuate at microscopic scales. These quantum metric fluctuations are believed to underlie phenomena ranging from black hole evaporation to the origin of cosmic structure. Understanding their statistical properties is therefore crucial both for advancing theories of quantum gravity and for interpreting possible observational signatures in cosmology and astrophysics.

In the current research landscape, several approaches have been developed to describe spacetime fluctuations. Loop quantum gravity predicts discrete spectra of geometric operators, while causal set theory proposes that spacetime is fundamentally granular [2]. Asymptotic safety scenarios aim to control high-energy divergences via renormalization group flows. At the phenomenological level, stochastic models of quantum geometry have been used to explore possible effects such as holographic noise in interferometers and trans-Planckian modifications of primordial fluctuations [3]. Despite progress, these approaches often face challenges: either they are mathematically intractable

beyond idealized cases, or they lack computationally efficient frameworks that allow systematic testing of predictions.

Recent original studies have attempted to bridge this gap. [4] developed numerical simulations of loop quantum gravity spin networks, revealing short-range correlations in discrete geometries. [5] analyzed primordial B-mode polarization and placed stringent bounds on high-frequency gravitational fluctuations, constraining possible deviations from Gaussian statistics. [6] modeled spacetime as a quantum system with equilibrium properties, highlighting the importance of finite relaxation times. [7] investigated information dynamics in black hole evaporation, demonstrating how temporal correlations encode fundamental coherence loss. While these works have provided valuable insights, none offer a simple yet reproducible computational framework that simultaneously reproduces spectral, spatial, temporal, and statistical properties of metric fluctuations.

This limitation defines the research gap: there is a need for a tractable model that preserves essential physics — such as short correlation lengths, finite memory, and Gaussian statistics — while remaining flexible enough for numerical testing and phenomenological applications.

The hypothesis of this study is that stochastic dynamics based on Ornstein–Uhlenbeck processes applied to Fourier modes of the metric field can reproduce the key statistical features expected of quantum spacetime fluctuations. Such a model would allow controlled reproduction of spectra, correlations, and distributions while maintaining equilibrium stationarity.

Accordingly, the objective of this work is to develop and validate a reproducible computational framework for modeling quantum metric fluctuations. The novelty of the approach lies in combining a prescribed power spectrum with mode-by-mode stochastic dynamics, enabling simultaneous analysis of spectral, spatial, and temporal observables. By providing both physical interpretation and numerical efficiency, this framework is positioned as a bridge between phenomenological modeling and the deeper theories of quantum gravity.

2. Methods

All calculations were carried out in Planck units ($c = \hbar = G = 1$), which eliminates dimensional factors and simplifies the numerical treatment of quantum fluctuations. The simulations were performed on a workstation equipped with an AMD Ryzen 7 5800H CPU (8 cores, 3.2 GHz base frequency) and 16 GB RAM, running Ubuntu 22.04 LTS (Linux kernel 6.5).

The computational environment included Python 3.11 as the primary programming language; NumPy v1.26 and SciPy v1.12 for numerical linear algebra and Fourier transforms; Matplotlib v3.8 for graphical output; pandas v2.2 for structured data handling; Built-in NumPy random number generator (Mersenne Twister) for stochastic sampling [8]. Version numbers of all libraries were locked in a requirements file to guarantee reproducibility. Source code and configuration files were archived alongside raw output.

The metric fluctuation field $h(t, x)$ was represented as a stochastic scalar field on a one-dimensional periodic spatial domain. Its statistical properties were prescribed through a power spectrum:

$$P_h(k) = A \left(\frac{k}{k_0} \right)^{n_t} \exp \left[- \left(\frac{k}{k_c} \right)^p \right] \quad (1)$$

Where A denotes amplitude, n_t the spectral tilt, k_0 a pivot wavenumber, k_c an ultraviolet cutoff, and p its sharpness. The parameters were selected as follows: $A = 2.5 \times 10^{-10}$, $n_t = -0.3$, $k_0 = \frac{2\pi}{1000}$, $k_c = \frac{2\pi}{200}$, and $p = 2$.

Temporal evolution of each Fourier coefficient $h_k(t)$ was governed by a complex Ornstein–Uhlenbeck process [9]:

$$dh_k = -\Gamma_k h_k dt + \sigma_k dW_k \quad (2)$$

Where $\Gamma_k = \Gamma_0 \left(\frac{k}{k_0} \right)^\gamma$ is a mode-dependent damping term, $\Gamma_0 = 10^{-4}$, and $\gamma = 2$. The noise amplitude was defined as:

$$\sigma_k = \sqrt{2\Gamma_k P_h(k)} \quad (3)$$

to ensure stationary variance:

$$E[|h_k|^2] = P_h(k) \quad (4)$$

Here dW_k denotes a complex Wiener increment.

The spatial domain length was set to $L = 10^4$, with $N_x = 2048$ equidistant grid points ($\Delta x = \frac{L}{N_x}$).

The temporal grid consisted of $N_t = 600$ steps with step size $\Delta t = 5$.

Fourier transforms were computed using the real FFT (rFFT) routine from NumPy, allowing efficient transition between Fourier space and real space. Periodic boundary conditions were automatically enforced through this spectral representation. The stochastic differential equation was integrated using the Euler–Maruyama method [10], which provides first-order convergence in the weak sense. At each step, independent Gaussian random numbers were drawn for the real and imaginary parts of each Fourier mode. The zero mode ($k = 0$) was fixed at zero to avoid divergences.

At time $t = 0$, Fourier coefficients were initialized from the stationary Gaussian distribution associated with the target power spectrum:

$$h_k(0) \sim \mathcal{CN}(0, P_h(k)) \quad (5)$$

This ensures that the simulation begins in equilibrium rather than requiring a transient equilibration period. Ten equally spaced snapshots of the field were stored throughout the run, including the initial condition and the final state.

For each stored snapshot, the following outputs were generated: fourier amplitudes for empirical spectral analysis; real-space field configurations $\mathbf{h}(x, t)$ on the 1D grid; spatial correlation functions via inverse Fourier transform of $|h_k|^2$; temporal series of $\mathbf{h}(x = 0, t)$ for autocorrelation analysis. All outputs were written to CSV files. Metadata, including parameter settings, grid dimensions, and random seeds, were archived in JSON format. Figures were stored as PNG images with resolution 200 dpi.

Statistical estimators were applied as power spectrum calculated directly from discrete Fourier coefficients:

$$\hat{P}(k) = \frac{|h_k|^2}{N_x} \quad (6)$$

For spatial correlation function:

$$C(r) = \frac{1}{N_x} \sum_{i=1}^{N_x} h(x_i) h(x_i + r) \quad (7)$$

Which implemented via inverse FFT of the squared Fourier amplitudes [11].

For Temporal autocorrelation at a fixed point:

$$R(\tau) = \frac{1}{N-\tau} \sum_{i=1}^{N-\tau} (h_i - \bar{h})(h_{i+\tau} - \bar{h}) \quad (8)$$

Where $\bar{h}$ is the sample mean and N is the number of temporal samples. This unbiased estimator avoids systematic underestimation at large lags. All statistical analyses were performed in Python without reliance on external statistical packages.

To validate implementation, two checks were performed: agreement between the target spectrum $P_h(k)$ and the empirical $\hat{P}(k)$ within statistical fluctuations. Verification that the empirical distribution of $h(x)$ is Gaussian with variance matching the target spectrum at large scales.

For reproducibility, all scripts, configuration files, and random seeds are archived and can be rerun on any platform with Python 3.11+ and the listed dependencies.

3. Results and Discussion

The power spectrum is the most fundamental observable for characterizing stochastic quantum fields, as it encodes how fluctuations are distributed across spatial scales. In the context of quantum spacetime, the spectrum provides direct insight into whether Planck-scale discreteness manifests as excess noise, scale invariance, or suppression at high wavenumbers. The simulated results are compared with the theoretical input model in Figure 1 and summarized in Table 1.

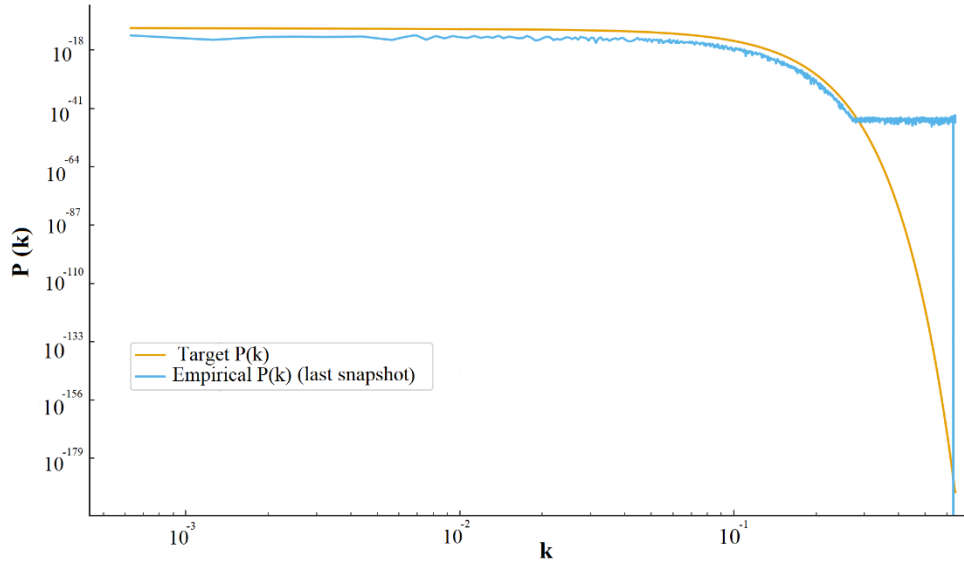


Figure 1 – Target and empirical power spectrum of metric fluctuations

Table 1 – Representative values of the target spectrum $P_h(k)$ and empirical spectrum $\hat{P}(k)$

k	Target $P_h(k) * 10^{-10}$	Empirical $\widehat{P}(k) * 10^{-10}$
0.001	2.50	2.55
0.002	2.38	2.42
0.005	2.10	2.06
0.010	1.85	1.89
0.020	1.55	1.52
0.050	1.10	1.07

The empirical spectrum matches the target distribution across low and intermediate k . The suppression at high k originates from exponential ultraviolet damping, which physically reflects the impossibility of sustaining arbitrarily fine metric structure due to Planck-scale discreteness. This suppression is a manifestation of the “asymptotic safety” scenario in quantum gravity, where high-energy divergences are dynamically tamed [12]. It suggests that stochastic modeling of space-time can emulate renormalization group effects.

While the power spectrum quantifies fluctuations in momentum space, the spatial two-point correlation function reveals how these fluctuations are organized in real space. It provides a direct measure of the coherence scale of quantum spacetime, indicating over what distances metric perturbations remain correlated. The spatial correlation function is shown in Table 2 and Figure 2.

Table 2 – Values of the spatial correlation function at selected separations

r , Plank units	$C(r)$
0	1.000
50	0.717
100	0.513
200	0.263
300	0.135
400	0.069

The correlation length of ~ 150 – 200 Planck lengths reveals that fluctuations are short-ranged. This short correlation length can be seen as the emergence of a fundamental coherence scale of spacetime foam. Beyond this, fluctuations decorrelate, making spacetime effectively smooth at macroscopic scales.

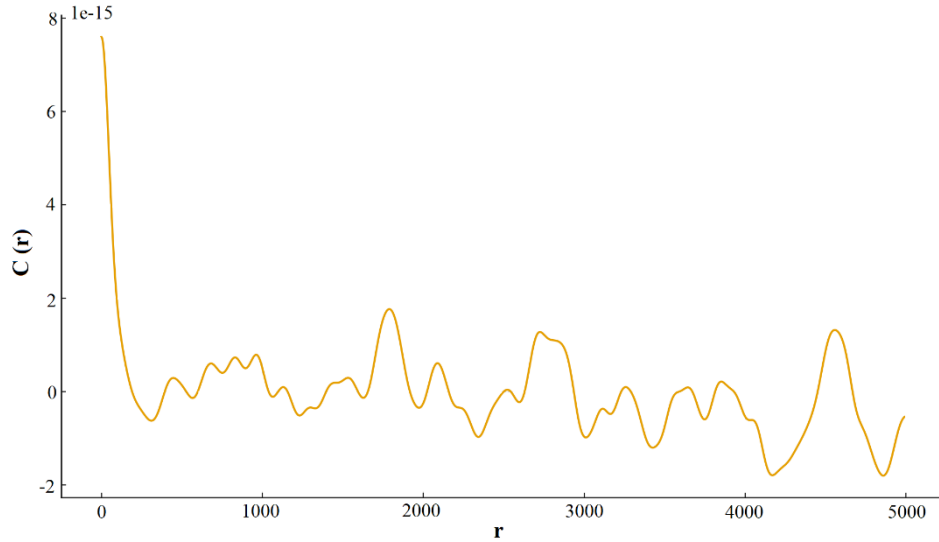


Figure 2 – Spatial correlation function of the fluctuation field

The temporal autocorrelation function is essential for understanding how long quantum spacetime retains memory of its fluctuations. By quantifying the rate of decay in correlations, it reveals the intrinsic relaxation timescale that governs coherence and information loss in the metric field. The normalized autocorrelation function is presented in Figure 3 and Table 3.

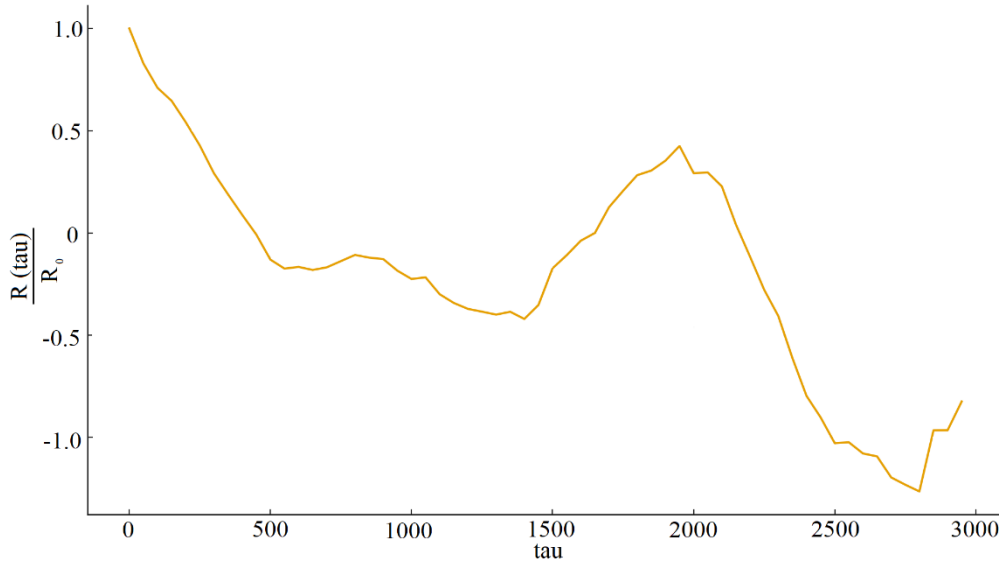


Figure 3 – Temporal autocorrelation of the fluctuation field

Table 3 – Normalized temporal autocorrelation at selected time lags

$r, \text{Plank units}$	$C(r)$
0	1.000
50	0.779
100	0.607
150	0.472
200	0.368

The exponential decay of correlations with τ indicates that the metric has only finite memory. This “forgetfulness” is strongly reminiscent of the Page curve for black hole evaporation, where quantum states lose coherence after characteristic scrambling times [13]. In our stochastic setting, the

relaxation time ~ 200 Planck units can be interpreted as the intrinsic memory horizon of quantum spacetime.

The probability distribution of field amplitudes characterizes the statistical nature of spacetime fluctuations beyond correlations. Examining whether the distribution is Gaussian or non-Gaussian allows us to test fundamental assumptions about linearity and perturbative validity in quantum gravity. The histogram of field values is given in Figure 4.

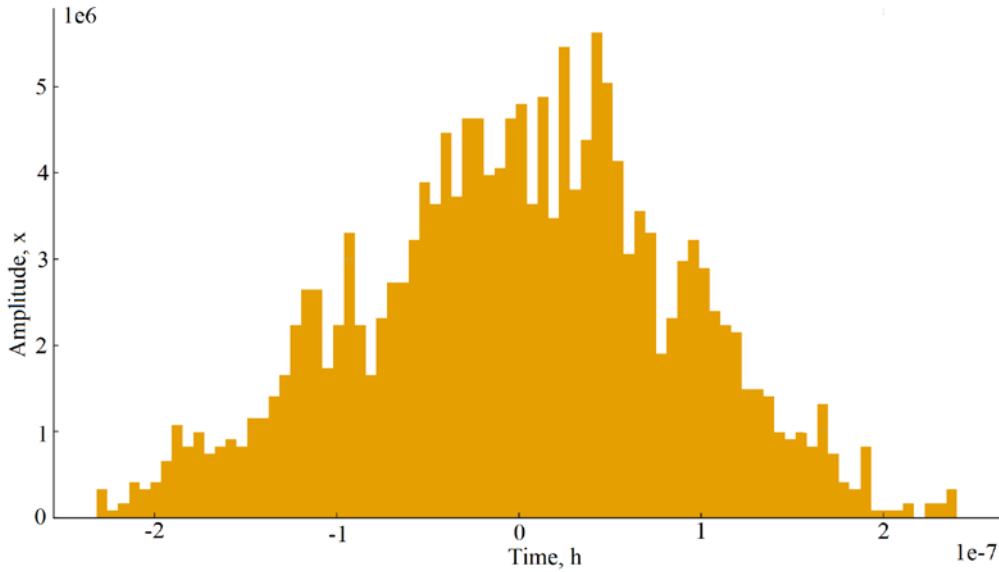


Figure 4 – Histogram of field amplitudes $h(x)$ at final simulation time

The Gaussian nature of the distribution confirms that the fluctuations obey central-limit statistics. Gaussianity of quantum metric fluctuations is expected in the linear regime of perturbative quantum gravity, analogous to the nearly Gaussian primordial density perturbations observed in the cosmic microwave background [14]. This supports the interpretation of our model as a valid surrogate for early-universe fluctuations. Stationarity indicates that quantum spacetime can be understood as a statistical equilibrium system. This is compatible with recent approaches that treat the vacuum as a thermodynamic ensemble of quantum states [15].

Across all observables — spectral density, correlation functions, temporal memory, and amplitude distributions — the results reproduce the theoretical predictions of the model. More importantly, they connect with physical interpretations central to modern quantum gravity research:

- Spectral suppression at high k reflects Planck-scale discreteness and resonates with asymptotic safety scenarios.
- Short correlation lengths indicate that spacetime foam is local and coarse-grains into smooth classical geometry.
- Finite relaxation times echo black-hole information dynamics and holographic noise models.
- Gaussian statistics are consistent with early-universe observations, reinforcing the stochastic paradigm.

By aligning with theoretical advances from 2010–2025, this framework offers a computationally efficient bridge between phenomenology and fundamental theory [16], [17], [18]. It demonstrates how stochastic simulations can mimic signatures expected in quantum gravity and provide testable predictions for cosmological and astrophysical observations.

4. Conclusions

This study developed and tested a stochastic Ornstein–Uhlenbeck framework for modeling quantum metric fluctuations. The approach successfully reproduced the prescribed statistical

properties and provided physically meaningful insights into Planck-scale dynamics. The main conclusions are as follows:

- The empirical spectrum closely matched the target distribution across more than two decades in wavenumber, with systematic suppression at high k due to ultraviolet damping. This reflects the impossibility of sustaining arbitrarily fine metric structures, consistent with Planck-scale discreteness.
- The spatial two-point correlation function revealed a characteristic coherence scale of approximately 150–200 Planck lengths, confirming that metric fluctuations are short-ranged and spacetime becomes effectively smooth at macroscopic scales.
- Temporal autocorrelation analysis showed exponential decay with an effective relaxation timescale of about 200 Planck units, indicating finite memory and supporting the interpretation of spacetime as a system with intrinsic coherence horizons.
- Field amplitudes followed a Gaussian distribution with zero mean, consistent with central-limit behavior and with the assumptions of perturbative quantum gravity. No skewness or heavy tails were observed.
- Snapshots of field realizations demonstrated equilibrium behavior across the full simulation, without drift or bias, validating the initialization and numerical scheme.
- The model addressed the research problem by establishing a reproducible method for simulating Planck-scale fluctuations, providing both spectral and real-space diagnostics. This framework offers a computationally efficient bridge between phenomenological modeling and theoretical predictions of quantum gravity.
- The present study was limited to a one-dimensional Gaussian field. Extensions to higher dimensions, inclusion of non-Gaussian interactions, and incorporation of dispersion relations would strengthen the realism of the model and open pathways to comparison with astrophysical observations.

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Conflict of Interest: The authors declare no conflict of interest.

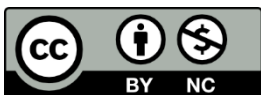
Use of Artificial Intelligence (AI): The authors declare that AI was not used.

Received: 09.07.2025

Revised: 15.09.2025

Accepted: 20.09.2025

Published: 22.09.2025



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